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ORIGINAL ARTICLE

Identification of Identical Bands and Energy Staggering Phenomena in Superdeformed Nuclei in A=190 Mass Region in the Framework of a Novel Proposed Simple Formula

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Abstract

A total of 11 superdeformed rotational bands (SDRB's) in the A = 190 mass zone were described in the framework of a novel proposed simple phenomenological rotational formula as outcome of collective model. The proposed model contains the original pure rotation limit AI (I+1) plus perturbation term proportionate to the cubic power of the spin I. The bandhead spins and the model parameters were selected by a best-fit method, utilizing a computerized search program to obtain the best match between experimental and calculated transition energies. The spin propositions are mostly consistent with the findings of earlier works. The calculated gamma-ray transition energies (Eγ) over spin (EGOS) in all SDRBs agree well with experimental data. Detailed assessment of the dynamic moments of inertia with rotational frequency is surveyed in detail which proves to be quite helpful for understanding the SDRB's properties such as identical bands (IBs). Four pairs namely: ^{191}Hg (SD2, SD3),^{193}Hg (SD3, SD4),^{194}Hg (SD2, SD3) and ^{193}Pb (SD3, SD4) are investigated as signature partners which exhibit $\triangle I = 1$ staggering effects their transition energies. We looked at this staggering by taking into account parameters that describe the variance between average transitions I+2→I→I-2 and the I+1→I-1 energies in its signature partner and a staggering parameter depends on the dipole transition energies connecting the two signature partners with that quadrupole transition energies. Large amplitude staggering is found. The bandhead moments of inertia of each signature partner pairs are found to be identical. Our calculations predicts the occurrence of $\triangle I = 2$ staggering effects in ^{194}Hg (SD1, SD2, SD3). This predicted $\triangle I = 2$ energy staggering has been inspected by computing parameters to indicate the finite difference approximation to the fourth-order derivative of that indicating energies. The phenomenon of IB is investigated for the nuclei ^{191}Hg and ^{192}Hg and their neighbors.

Keywords: Collective rotational model, Energy staggering, Identical bands, Signature partner, Superdeformed nuclei

1. Introduction

The first superdeformed (SD) nucleus was revealed in 1986 in ^{152}Dy [1]. Now the rapid experimental progress on high spin states using the high-resolution gamma(γ)-ray multidetector arrays leads to the discovery of an abundance of superdeformed rotational bands (SDRB's) in sundry nuclear mass ratios ranging from A~ 40 to A~ 240 [2]. Particularly, extensive experimental data was gained in the A~ 190 region. A characteristic feature in this region is that the SDRBs were detected at a bit low spin and extended to ~40ħ. The majority of the A 190 SD bands show the same consistent upward trend in the dynamical moment of inertia $J^{(2)}$ as the rotational frequency (ħω) increases. Due to the sequential alignment of a specific pair of high j (low Ω) intruder $i_{13/2}$ protons and $j_{15/2}$ neutrons and from the gradual disappearance of pairing correlations with the collective rotation [3–5].

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One of the most interesting and unpredicted feature discovered in SD nuclei is the presence of identical bands (IBs) in several pairs of bands be owned by neighboring nuclei with different mass number. Throughout a protracted series of conversions, γ-ray transition energies of the SD bands in deferent nuclei can be similar within a few keV. The IBs in the two nuclei must have identical dynamical moments of inertia $J^{(2)}$ and identical alignment over a large number of conversions. The first IB's were detected in the A~150 region, which include the (^{152}Dy, ^{151}Tb*) and (^{151}Tb, ^{150}Gd*) pairs [6,7]. Soon after, examples were reported around A~190 [8,9]. Now many cases of IB pairs are found both in normal and superdeformed regions [10,11]. Several groups attempted to understand the phenomenon of IBs in the framework of phenomenological and semi-phenomenological methods [12−16]. The incremental alignment △i is introduced by Stephens [7,9] to compare the transition energies in SD band of interest directly to other reference bands. The benefit of this incremental alignment is that it may be used without any prior knowledge of the spins of the states. Transition energies that are almost similar produce △i values that are extremely close to zero.

Another interesting feature occur in the SD nuclei is the detection of △I = 2 staggering in γ-ray transition energies. Rotational sequences in SDRB's with nuclear spins differing by 2ħ split into two branches with spin values I, I+4, I+8, and I+2, I+6, I+10, respectively [17−19].

This is also called the △I = 4 bifurcation. The bifurcation amplitudes are very small (about 0.3 keV). The occurrence of a regular staggering pattern in SD bands was discussed as potential proof for fourfold symmetry, the $C_{4\pi}$ symmetry in the intrinsic Hamiltonian of the geometric model [20−23]. Phenomenologically it can be proved that the appearance of △I = 2 staggering maybe linked to the occupation of high J intruder orbits from the N = 6 and N = 7 oscillator shells and that the intrinsic hexadecapole moments of specific single particle states may along had a part in producing the observed △I = 2 staggering [17,24]. Also the △I = 2 staggering effect was interpreted in many ways [25,26]. Lately the △I = 2 staggering was shown in the ground bands of normally deformed (ND) nuclei like thorium nuclei [27].

There is another kind of staggering that happen in signature partner pairs of SDRB's called the △I = 1 staggering [28−31]. Most of SDRB's detected in odd-A and odd−odd nuclei in the mass region A ~190 are signature partner pairs, each pair present a large amplitude △I = 1 staggering [32,33] and also the bandhead moments of inertia of each pair are the same. The magnetic dipole transition M1 connecting the two signature partners was observed experimentally in ^{193}Tl [29] and ^{193}Hg [34]. The spin assignments for most of SDRB's illustrate the most difficult and unsolved problem. The absolute spins, parities and excitation energies of SD bands are unknown and have not been experimentally determined with the exception that the γ-ray decays from SD to low lying normal deformed states have later been noted in ^{194}Hg (SD1, SD3) and ^{194}Pb (SD1) enabling the level spins in the SD nuclei to be assigned [35−38].

The main purposes of the present work are: (i) To develop the collective rotational model to propose a new novel formula for numbering the γ-ray transition energies and level spins in SD bands in the mass region A~190. (ii) To study the variations of dynamic $J^{(2)}$ moments of inertia with increasing the rotational frequency ħω. (iii) To investigate the △I = 1 and △I = 2 staggering in transition energies. (iv) To search for identical bands.

2. Model and procedure

A very helpful model of nuclear structure for a nucleus which has a structure away from the closed shell type is the rotational model. In this model the excitation spectrum shows the typical rotational design [39].

$$E(I) = \frac{\hbar^2}{2J} I(I+1) \tag{1}$$

where I is the rotational angular momentum and J signify the moment of inertia about the rotational axis. For rotational nuclei, the data on the first excited states of even−even nuclei led directly to the definition of an experimental moment of inertia

$$J = \frac{3\hbar^2}{E(2_1^+)} \tag{2}$$

a similar moment of inertia for the odd nuclei needs the additional experimental determination of the spin of the excited state. Although the moment of inertia can be determined experimentally, it cannot directly determine by the rotational model because the moment of inertia J is not always constant, as would be expected from the rotator formula (1).

In this paper, we may try to increase the early formula equation (1) by adding second term as perturbation, to reach a new energy expression as a first time formula

$$E(I) = \frac{\hbar^2}{2J} I(I+1)[1+(a+bI)] \quad (3)$$

we can treat J, a, b as the three model parameters. In superdeformed (SD) nuclei the measurements made via experimentation are the γ-ray transition energies $E_\gamma(I)$ between levels differing by two units of angular momentum, then one can note the transition energy as:

$$E_{\gamma 2}(I) = E(I) - E(I-2)$$

$$= \frac{\hbar^2}{2J}\left[(1+a)(4I-2) + 2b\left(3I^2 - 4I + 2\right)\right] \quad (4)$$

$$E_{\gamma 2}(I) = A\left(C_2 I^2 + C_1 I + C_o\right) \quad (5)$$

with $A = \frac{\hbar^2}{2J}, C_o = -2 - 2a + 4b, C_1 = 4 + 4a - 8b, C_2 = 6b$.

3. Approximate estimation of bandhead spin and model parameters

If we clarify the transition energy ratio R as

$$R = \frac{E_{\gamma 2}(I_o + 4)}{E_{\gamma 2}(I_o + 2)} \quad (6)$$

where I_o is the bandhead spin, therefore using equation (1), yield

$$R = \frac{4I_o + 14}{4I_o + 6} \quad (7)$$

this determine the bandhead spin I_o as a first estimation before using the fitting procedure, the value of I_o is given by [40].

$$I_o = \frac{1}{4}\left[\frac{8E_\gamma(I_o+2)}{E_\gamma(I_o+4) - E_\gamma(I_o+2)} - 6\right] \quad (8)$$

The corresponding value of the moment of inertia J becomes.

$$J = \frac{4I_o + 6}{E_{\gamma 2}(I_o + 2)} \quad (9)$$

Now, to estimate the model parameters a, b, we consider the two transition energies

$$e_2 = E(I+2 \rightarrow I) = E(I+2) - E(I) \quad (10)$$

$$e_4 = E_\gamma(I+4 \rightarrow I) = E(I+4) - E(I) \quad (11)$$

Substituting for E from equation (3), yields

$$e_2 = A(4I+6)(1+a) + Ab\left(6I^2 + 16I + 2\right) \quad (12)$$

$$e_4 = A(8I+20)(1+a) + Ab\left(12I^2 + 56I + 80\right) \quad (13)$$

Solving to get a, b yield

$$A(1+a) = (1/\triangle)\left[\left(6I^2 + 28I + 40\right)e_2 - \left(3I^2 + 8I + 6\right)e_4\right] \quad (14)$$

$$Ab = (-1/\triangle)\left[(4I+10)e_2 - (2I+3)e_4\right] \quad (15)$$

$$\text{with } \triangle = 24\left(I^2 + 5I + 5\right) \quad (16)$$

We can also determine the model parameters as a first estimation by another general method, by considering all the experimental transition energies Eγ(I) equation (4) by putting α = A (1 + a) and β = Ab, then yield

$$\left[E_{\gamma 2}(I)/(I - ½)\right] = 4\alpha + \left[6I^2 - 8I + 4 / (I - ½)\right]\beta \quad (17)$$

The plot of $E_\gamma(I)/(I - ½)$ against $(6I^2-8I+4)/(I - ½)$ give a straight line of slope β and intersect 4α. That is the two parameters a, b are determined as a first estimation.

4. Rotational frequency and moments of inertia

The deviations from the rotational energy equation (1) may be showed as a dependence of the moment of inertia J on the rotational frequency (ħω). The moment of inertia J is defined as the ratio of the angular momentum ($\hbar\hat{I}$) to the angular frequency (ω);

$$J = \hbar\frac{\hat{I}}{\omega}, \hat{I} = \sqrt{I(I+1)} \quad (18)$$

The ħω is obtained from the canonical equation,

$$\hbar\omega = dE/d\hat{I} \quad (19)$$

We thus obtain

$$I = \frac{\hbar^2}{2}\left(\frac{dE}{d\hat{I}}\right)^{-1} \quad (20)$$

For SD bands, γ-ray transition energies are the only spectroscopic data available. In order to compare the SD bands' structures, information regarding their γ-ray energies was typically converted into values for the dynamical moment of inertia.

$$J^{(2)} = \frac{d\hat{I}}{d\omega} \cong \frac{\Delta I}{\Delta\omega} = \frac{2}{\Delta\left(\frac{1}{2}E_\gamma\right)} = \frac{4}{\Delta E_\gamma} \quad (21)$$

Where $\triangle E_\gamma$ is the variation between two successive γ-ray energies in the cascade,

$\triangle E_{\gamma} = E_{\gamma}(I+2) - E_{\gamma}(I)$

For the discrete experimental spectra, equation (1) and equation (21) may be used to produce a value of the kinematic moment of inertia

$$J^{(1)} = (2I-1) / E_{\gamma}(I \rightarrow I-2) \quad (22)$$

5. Identical bands

The discovery that multiple distinct nuclei with various mass numbers can have SD bands with almost similar transition energies to within an average of around 1−3 keV over huge spin periods is one of the more interesting results of the research of SD nuclei. Because the transition energy is approximately twice as the rotational frequency, the rotational frequencies of the two bands must be quite comparable, which suggests that the dynamical moments of inertia are almost identical. The initial discovery was that a SD (β*e* = 0.6) in ^{151}Tb possessed a band of levels whose transition energies were almost identical to those of an SD band in the isotone. A band of levels whose transition energies were essentially equal to those of a SD band in the isotone ^{152}Dy [6].

As an example in the A~ 190 region, it is discovered that over a spin range of 20ħ, the transitions of an excited band in ^{194}Hg are within roughly 1 keV of those of the yrast band in ^{192}Hg. In comparison to A150, A190 contains significantly more instances of identical SD bands.

The origin and the abundance of the identical bands phenomenon were investigated, but no definitive interpretation had appeared. This phenomenon had great deal of attention [12−16]. To compare between SD bands, Stephens [7] proposed the incremental alignment $\triangle i$ defined by:

$$\Delta i = 2\frac{\Delta E_{\gamma}}{\Delta E_{\gamma}^{ref}} \quad (23)$$

where $\triangle E\gamma$ is determined by deducting the transition energy in the interest band A from the nearest transition energy in the reference SD band B, and ΔE_{γ}^{ref} is determined as the difference in energy between the two nearest transitions in the SD bands of reference B, which is

$$\Delta i = \frac{E_{\gamma}^{A}(I+2) - E_{\gamma}^{B}(I)}{E_{\gamma}^{B}(I+2) - E_{\gamma}^{B}(I)} \quad (24)$$

where A is the target nucleus and B is the reference nucleus.

Staggering of the transition energies for SDRB's

The $\triangle I = 2$ staggering in transition energies of SD bands shows that there is a proportionate shift between the levels with angular momentum I, I+4, I+8, and the levels with angular momentum I+2, I+6, I+10, …The $\triangle I = 2$ staggering effect (or $\triangle I = 4$ bifurcation) in transition energies of some SD bands had attracted much interest. To make this effect simple, the energy difference $\triangle E_{\gamma}$ between two successive γ-ray transitions after the subtraction of a smooth reference must be calculated which can be written as

$$\delta(\Delta E_{\gamma}) = \Delta E_{\gamma} - \Delta E_{\gamma}^{ref} \quad (25)$$

Using the notation of Flibotte et al. [17], (four pint formula) for the ΔE_{γ}^{ref} , we have

$$\delta(\triangle E_{\gamma}(I)) = (1/8)\left[E_{\gamma}(I+2) - 3E_{\gamma}(I) + 3E_{\gamma}(I-2) - E_{\gamma}(I-4)\right] \quad (26)$$

where $E_{\gamma}(I)$ is the change from an I spin state to an I-2 spin state.

If we give the definition of Cederwall et al. [18], (five point formula|) the difference is given by;

$$\delta(\triangle E_{\gamma}(I)) = (1/16)\left[E_{\gamma}(I+4) - 4E_{\gamma}(I+2) + 6E_{\gamma}(I) - 4E_{\gamma}(I-2) + E_{\gamma}(I-4)\right] \quad (27)$$

The rotational energies in some SD bands, are irregular and the δ ($\triangle E_{\gamma}$) reveals a zigzagging pattern between adjacent spin states. To observe small differences in the transition energies, we introduce the staggering parameter S (I) such as [41],

$$S(I) = \delta(\triangle E_{\gamma}) \exp - (\triangle E_{\gamma})cal \quad (28)$$

Another kind of staggering was observed in signature partner pairs of SDRB's in odd−odd and odd - mass nuclei. This is the $\triangle I = 1$ staggering, it is a shift of energy levels with spin I in a band and energy levels with spin I±1 in another band of the signature partner. Most of these signature partners in the mass region A~190 demonstrate a large amplitude staggering and each pair's bandhead moments of inertia are very identical.

In order to explore the $\triangle I = 1$ staggering in signature partner pair of bands, one must educe the difference between the average transitions I +2→I→I −2 energies in one band and the transition I +1→I −1 energies in the signature partner [28−31].

$$\triangle^2 E_\gamma(I) = \left[(½)\left(E_\gamma(I+2)+E_\gamma(I)\right) - E_\gamma(I+1)\right]$$

$$= (¼)\left[E_\gamma(I+2) - 2E_\gamma(I+1) + E_\gamma(I)\right] \quad (29)$$

In previous works [32,33] the $\triangle I = 1$staggering was investigated by considering a staggering function Y(I) rely on the dipole γ-ray transitions connecting the two signature partners and the quadruple transitions in each band, Y(I) possesses the form:

$$Y(I) = (2I-1)/I)$$
$$[(E(I)-E(I-1))/(E(I)-E(I-2))] - 1$$
$$= (2I-1)/I)\left[E_{\gamma 1}(I)/E_{\gamma 2}(I)\right] - 1 \quad (30)$$

where

$$E_{\gamma 1}(I) = E(I) - E(I-1)$$

$$= (Ab - A'b')I^3 + [A(1+a+b) - A'(1+a'-2b')]I^2 +$$

$$[A(1+a) + A'(1+a'-b')]I \quad (31)$$

and $E_{\gamma 2}(I) = E(I) - E(I-2)$.

$$= (A'b' - Ab)I^3 + [A'(1+a'+4b') - A(1+a+b)]I^2 +$$

$$[A'(3+3a'+5b') - A(1+a)]I + A'(2+2a'+2b')/I \quad (32)$$

6. Results and discussion

The data set in the present work contain in total 11 SDRB's, three SDRB's unveiled $\triangle I = 2$ staggering in their E_γ transition energies and four pairs represent signature partners exhibit $\triangle I = 1$ staggering. The model parameters (J, a, b) and the level spins have been adopted by using a simulated search program. The sensitivity of fitting is taken according to the common transition energy root-mean-square (rms) deviations given by

$$\chi = \left[\frac{1}{N}\sum_{n=1}^{N}\left|\frac{E_\gamma^{exp}(I_i) - E_\gamma^{cal}(I_i)}{E_\gamma^{exp}(I_i)}\right|^2\right]^{\frac{1}{2}}$$

where N represents how many data points are included in the fitting process. The experimental data were derived from Reference [2].

The spin of the bandhead I_o is taken as the closest half integer of the fitted I_o, then another fit with only (J, a, b) as free parameters is made. The best model parameters (J, a, b) and the predicted band head spin I_o for each band are outlined in Table 1.

The lowest transition energies $E_\gamma(I_o + 2 \rightarrow I_o)$ and the rms deviation χ are also indicated in the table. The present assigned spins of our selected SDRB's are quite consistent with that estimated values in previous works [16,26,32,33]. The calculated γ-ray transition energy/spin EGOS = E_γ (I)/I [42], are plotted in Fig. 1 and compared with experimental data. Theory and experiment show excellent agreement, which provides strong support for the hypothesis.

Using the suggested level spins and the adopted model parameters, the rotational frequency ħω and the systematic features of the dynamic moments of inertia $J^{(2)}$ of our selected SDRB's are extracted and plotted ageist ħω in Fig. 2a,e. For the isotones N = 111, 112, 113, 114, and ^{194}Hg (SD1, SD2, SD3). We notice that $J^{(2)}$ of all SDRB's change smoothly increasing with increasing ħω, this smooth increase is credit to the successive alignment of N = 6 ($i_{13/2}$) protons and N = 7 ($j_{15/2}$) neutrons in the presence of pairing. Clearly the $J^{(2)}$ values for the excited SD bands are very close to the yeast SD bands in their Z+1 isotones.

In some SD bands, the $\triangle I = 2$ staggering is corresponding to a shift of states with spins I, I+4, I+8,

Table 1. The calculated model parameters (J, a, b) obtained from the fitting procedure and the proposed bandhead spin I_o for the selected superdeformed rotational bands in A~190 mass region. The experiment lowest transition energies $E_\gamma(I_0 + 2 \rightarrow I_0)$ for every superdeformed band are also given [2].

Band	E_γ [keV] $(I_0 + 2 \rightarrow I_0)$	I_0 [ħ]	J (ħ^2 MeV^{-1})	a	b (x 10^{-3})	χ
^{191}Hg(SD2) (SD3)	252.4	10.5	123.7660	0.34253	−2.27628	0.0694
	272.0	11.5	128.2192	0.33307	−2.64517	0.0593
^{192}Hg(SD1)	214.4	8.0	132.3931	0.52706	−3.65237	0.2088
^{193}Hg(SD3) (SD4)	233.5	9.5	128.3410	0.40471	−2.73102	0.0896
	254.0	10.5	127.8321	0.39789	−2.68071	0.1158
^{194}Hg(SD1) (SD2) (SD3)	211.7	8.0	135.1482	0.55470	−3.86490	0.5028
	200.8	8.0	128.8194	0.40758	−2.75224	0.0766
	222.0	9.0	127.7031	0.38856	−2.61199	0.0866
^{193}Pb(SD3) (SD4)	251.5	10.5	118.9281	0.27978	−1.82787	0.0569
	273.0	11.5	129.4900	0.41477	−2.80555	0.0224
^{194}Pb(SD1)	124.9	4.0	130.4811	0.50014	−3.44693	0.1060

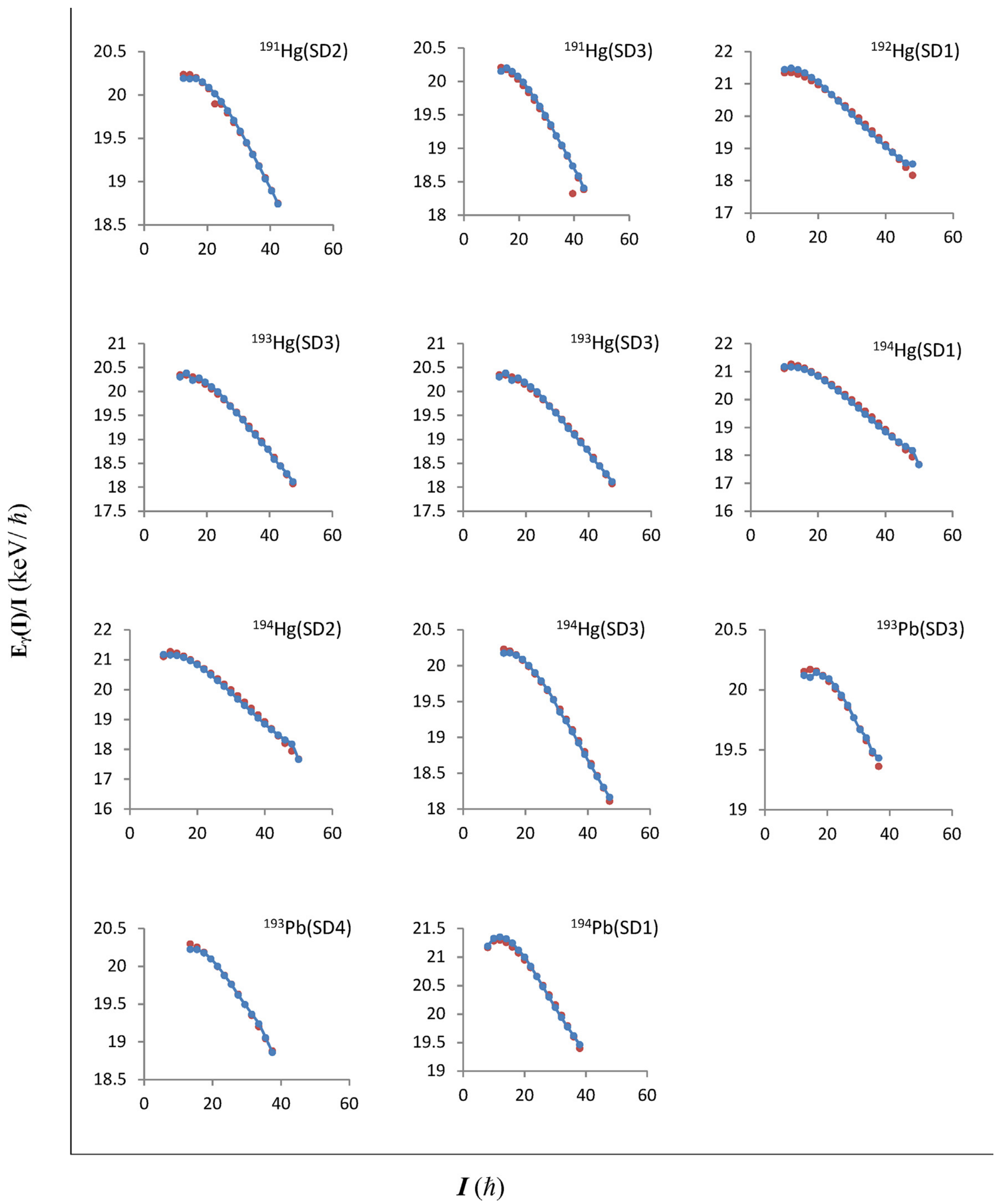

Fig. 1. The calculated γ-ray transition energy energies gamma-ray over spin (I) versus spin I. The selected superdeformed rotational bands are compared with experimental values [2]. Solid curves represent theoretical calculations while closed circles represents experimental values.

relative to states with spins I+2, I+6, I+10,. The presence of a $\triangle I = 2$ staggering in the γ-ray transition energies of the three SDRB's ^{194}Hg (SD1, SD2, SD3) for each band can be exhibited by calculating the fourth derivative of the γ-ray transition energies $S^{(4)}$ (I) at a given spin defined in equation (28) and plotted against the rotational frequency (ħω) as shown in Fig. 3. A significant staggering with large amplitude is detected for all chosen SDRB's. Four pairs of signature partners SD bands are proposed. One pair in even–even ^{194}Hg nucleus and three pairs in odd-A ^{191,193}Hg, ^{193,195}Pb nuclei. The $\triangle I = 1$ energy staggering found in these nuclei are examined by calculating the staggering parameters $\triangle^2 E_\gamma$

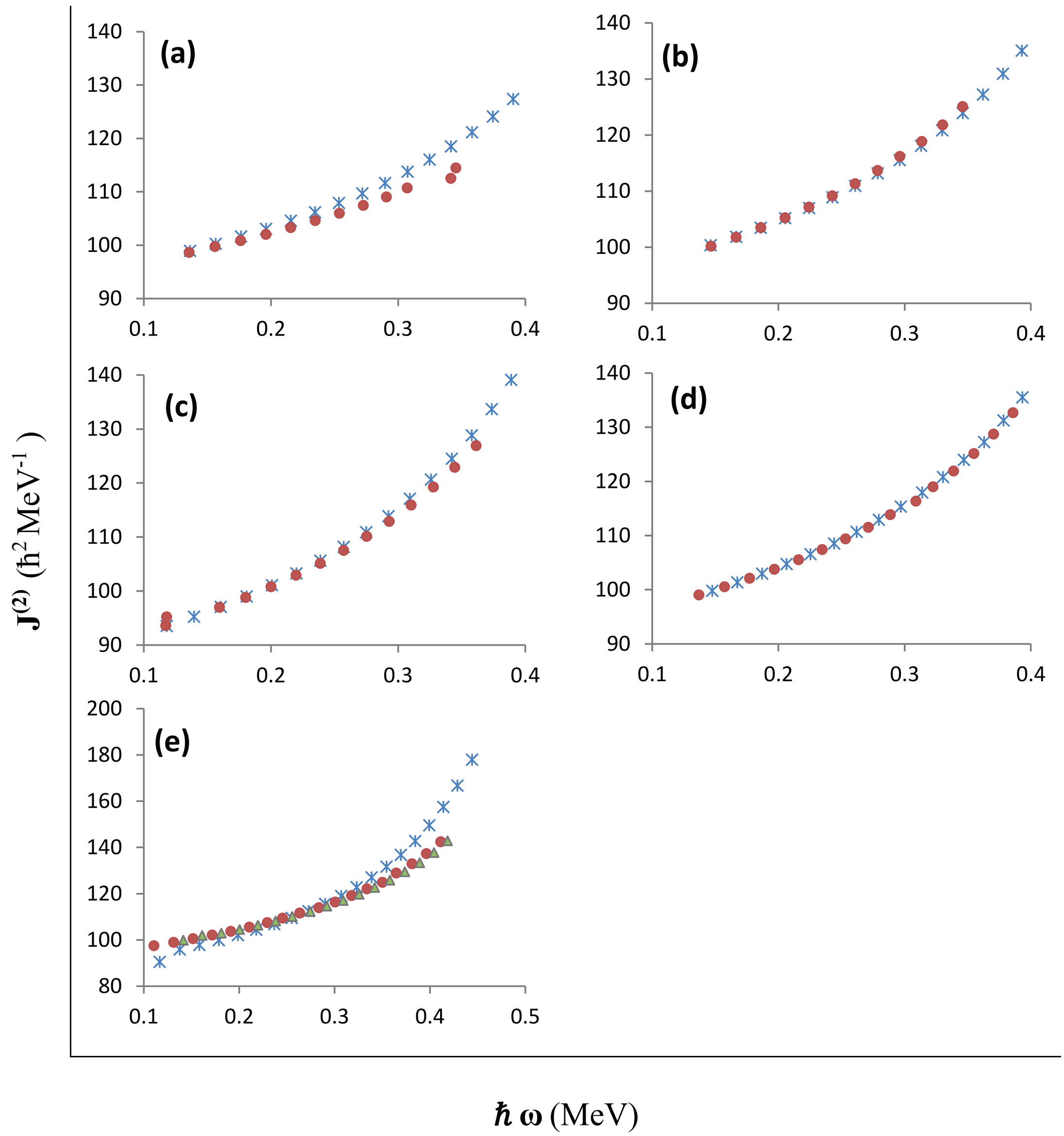

Fig. 2. The calculated dynamical moment of inertia $J^{(2)}$ versus rotational frequency $\hbar\omega$ for (a) The isotones N = 111 (^{191}Hg (SD2)), ^{193}Pb (SD3), (b) The isotones N = 111(^{191}Hg (SD3)), ^{193}Pb (SD4), (c)The isotones N = 112 (^{192}Hg (SD1)), ^{194}Pb (SD1), (d) The isotones N = 113 (^{193}Hg (SD4)), ^{193}Pb (SD3), (e) The three bands of ^{194}Hg (SD1, SD2, SD3).

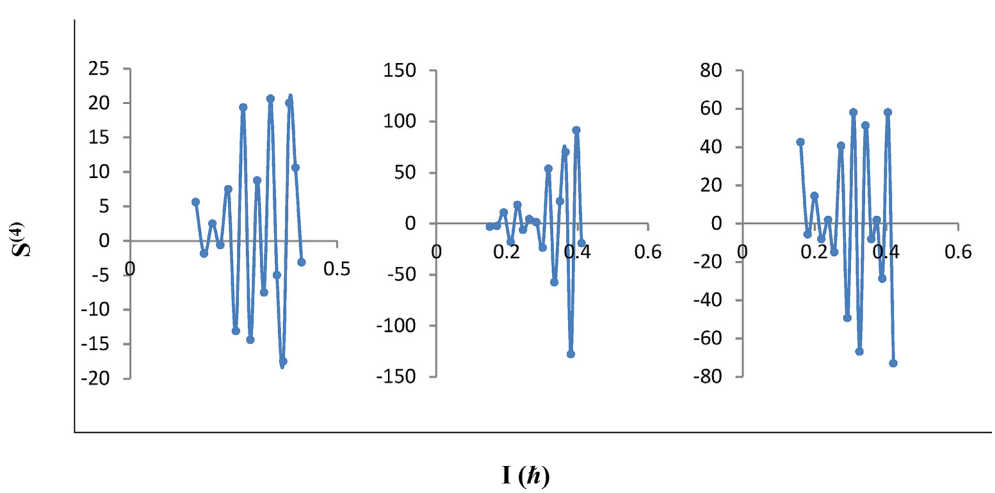

Fig. 3. Calculated $\Delta I = 2$ staggering parameter $S^{(4)}$ versus nuclear spin I for ^{194}Hg (SD1, SD2, SD3).

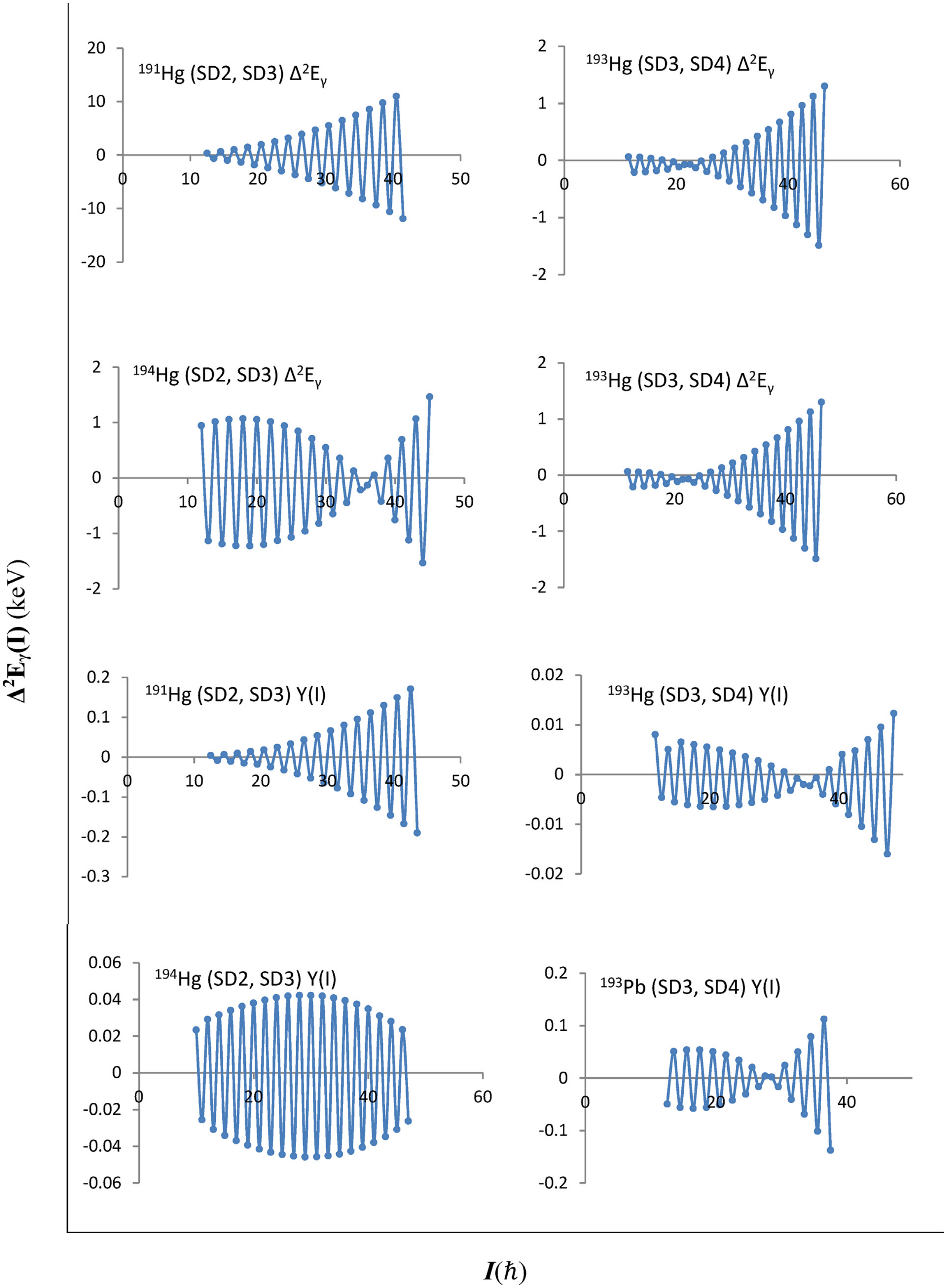

Fig. 4. The energy staggering parameter $\Delta^2 E_\gamma(I)$ as function of nuclear spin(I) for the four pairs signature partners in Hg and Pb nuclei.

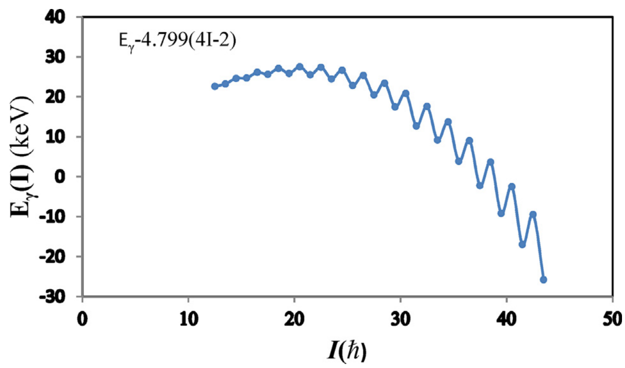

Fig. 5. ΔI = 1 staggering for the signature partner pairs ^{191}*Hg (SD2, SD3). The calculated transition energies as a function of the nuclear spin I to exhibit.*

(I) and Y (I) equations (29) and (30) as functions of spin. The staggering parameters $\triangle^2E_\gamma$ (I) depends on the average transitions I+2→I and I→I −2 energies in one band and the transition I+1→I−1 energy in the signature partner.

The staggering parameter Y (I) depended on the dipole transition energies E_γ (I→I −1) connecting the signature partners and the quadruple transition energies E_γ (I+2→I) within each band. The calculated values of $\triangle^2E_\gamma$ (I) and Y (I) are plotted versus spin I as presented in Fig. 4. It is seen that; all signature partners show zigzag behavior with large amplitude staggering.

The $\triangle I = 1$ staggering also happen if we subtract a rigid reference from the transition energies E_γ (I). This is illustrated in Fig. 5 for the pair ^{191}Hg (SD2, SD3) as an example.

To better understand the identical bands (IBs) phenomenon, the different in the transition energies $\triangle E_\gamma$ between the yeast SD band transitions in ^{192}Hg and the corresponding yeast bands in ^{194}Hg and ^{194}Pb against the transition energy E_γ are shown in Fig. 6a. We see that up to ħω ~0.25 MeV, the $\triangle E_\gamma$ values are small and constant for ^{192}Hg (SD1) and ^{194}Pb (SD1), while $\triangle E_\gamma$~ 4 keV for ^{192}Hg (SD1) and ^{194}Hg (SD1). Therefore, the yeast SD bands in the

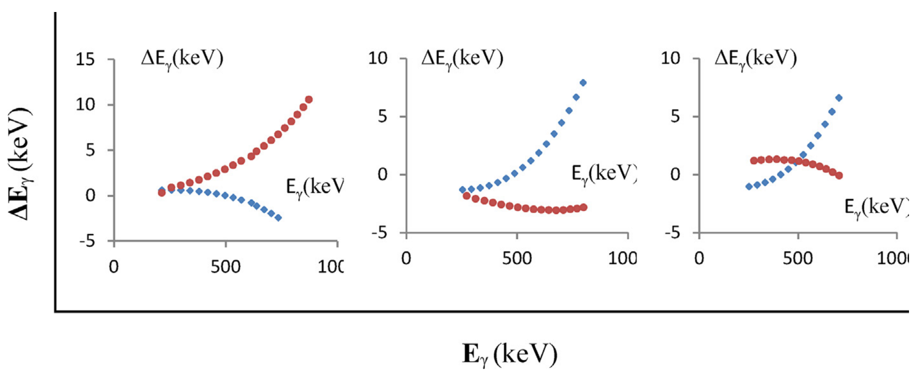

Fig. 6. The calculated differences in γ-ray transition energies ΔE_γ versus E_γ for the following identical SD bands (a) ^{192}*Hg (SD1)-*^{194}*Pb (SD1),* ^{192}*Hg (SD1)-*^{194}*Hg (SD1), (b)* ^{191}*Hg (SD2)-*^{193}*Hg (SD4),* ^{191}*Hg (SD3)-*^{193}*Hg (SD3) and (c)* ^{193}*Pb (SD3)-*^{191}*Hg (SD2),* ^{193}*Pb (SD4)-*^{191}*Hg (SD3).*

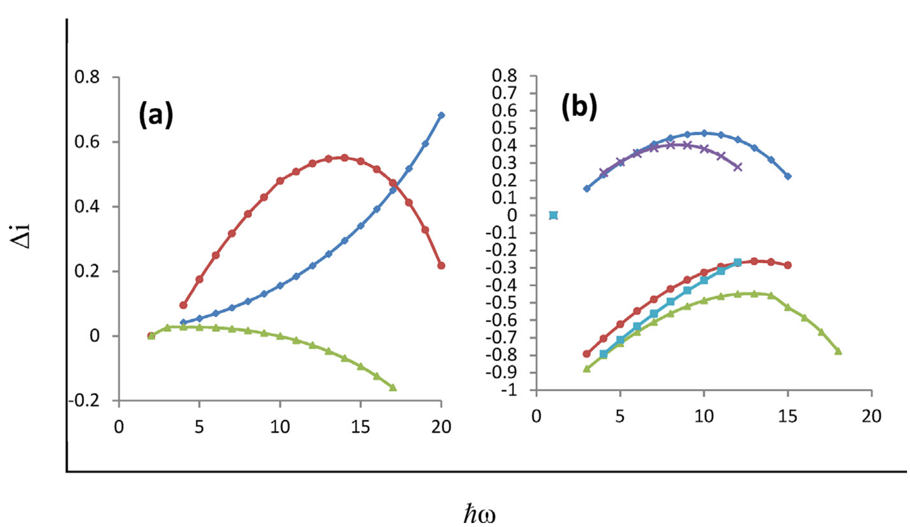

Fig. 7. The incremental angular momentum Δi versus rotational frequency ħω for (a) ^{194}*Hg (SD1),* ^{193}*Hg (SD4) and* ^{194}*Pb (SD1) relative to* ^{192}*Hg (SD1) as a reference (b)* ^{191}*Hg (SD2, SD3),* ^{193}*Hg (SD3) and* ^{193}*Pb (SD3, SD4).*

two isotones (N = 112) ^{192}Hg (SD1) and ^{194}Pb (SD1) are considered as IB's, this similarly suggested that the added protons on ^{194}Pb do not change the SD rotational properties in the observed frequency range. On the other hand, the two bands ^{192}Hg (SD1) and^{194}Hg (SD1) are too large to consider these two bands identical ones. Also additional information about IB's, we plotted in Fig. 6b, c, $\triangle E_{\gamma}$ versus E_{γ} for exited bands in ^{191}Hg, ^{193}Hg, ^{193}Pb. Both SD bands in ^{191}Hg (SD2, SD3) and ^{193}Hg (SD4, SD3) have nearly identical transition energies in the frequency range 0.13 MeV $\leq \hbar\omega \leq$ 0.34 MeV. The γ transition energies in ^{193}Pb (SD3, SD4) are identical to the ones SD2 and SD3 in the isotones (N = 111) ^{191}Hg (SD2, SD3).

The incremental alignment $\triangle i$ is calculated and plotted against the rotational frequency $\hbar\omega$ in Fig. 7 for the above SDRB's to choose the IB's with the yeast band in ^{192}Hg as a reference. In general the calculated $\triangle i$ are shown to cluster around the values $\triangle i = \pm 1$, 0.5 and 0 for $\hbar\omega \geq$ 0.2 MeV.

7. Conclusion

In this study, we suggested a model with a perturbation term proportional to the spin's cubic power along with the original pure rotator component. Eleven SDRB's in nuclei Hg, and Pb of the mass region A~190 are considered. The bandhead spins and the model parameters have been extracted by fitting the calculated transition energies E_{γ}^{cal} with the experimental ones E_{γ}^{exp} utilizing a computer-based simulated search program. The rotational frequency $\hbar\omega$, the dynamic $J^{(2)}$ moment of inertia are calculated. The systematic variation of $J^{(2)}$ with $\hbar\omega$ is investigated. The SDRB'S ^{194}Hg (SD1, SD2, SD3) show $\triangle I = 2$ staggering effects in their γ-ray transition energies by applying a staggering parameter shown the divergence of the γ-ray energies from smooth reference characterizing the finite difference approximation to the fourth order derivative of the γ-ray transition energies at a given spin (Cedeawall five point formula). To exhibit the $\triangle I = 1$ staggering in signature partner pairs of ^{191,193,194}Hg, and ^{193}Pb we determined the variations between the average transitions $I+2 \rightarrow I \rightarrow I-2$ energies in one band and the transition $I+1 \rightarrow I-1$ energies in its signature partner. Also A staggering parameter depends on the dipole transitions linking the signature partners. The phenomenon of identical bands is investigated for ^{191,192}Hg and their neighbors by calculating the differences between their γ-ray transition energies; also the incremental alignment has been calculated.

Conflicts of interest

The authors have no conflicts of interest to declare.

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