[Al-Azhar Bulletin of Science](https://absb.researchcommons.org/journal)

Volume 34 | Issue 3 Article 3

2023

Section: Mathematics and Statistics

BAYESIAN AND NON-BAYESIAN ESTIMATION BASED ON STEP STRESS-PARTIALLY ACCELERATED LIFE TESTING FOR ODD GENERALIZED NADRAJAH HAGHIGHI EXPONENTIAL DISTRIBUTION

Sarah Mohammad Behairy
Department of Statistics, Faculty of Commerce, AL-Azhar University (Girls' Branch), Cairo, Egypt

Neama Taher Al-Sayed
Department of Statistics, Faculty of Commerce, AL-Azhar University (Girls' Branch), Cairo, Egypt, neamaalsayed2452.el@azhar.edu.eg

How to Cite This Article

Behairy, Sarah Mohammad and Al-Sayed, Neama Taher (2023) "BAYESIAN AND NON-BAYESIAN ESTIMATION BASED ON STEP STRESS-PARTIALLY ACCELERATED LIFE TESTING FOR ODD GENERALIZED NADRAJAH HAGHIGHI EXPONENTIAL DISTRIBUTION," *Al-Azhar Bulletin of Science*: Vol. 34: Iss. 3, Article 3.

DOI: https://doi.org/10.58675/2636-3305.1652

BAYESIAN AND NON-BAYESIAN ESTIMATION BASED ON STEP STRESS-PARTIALLY ACCELERATED LIFE TESTING FOR ODD GENERALIZED NADRAJAH HAGHIGHI EXPONENTIAL DISTRIBUTION

Cover Page Footnote

The authors are grateful to the referees and the associate editor for their valuable comments which resulted in an improved paper.

ORIGINAL ARTICLE

Bayesian and Non-Bayesian Estimation Based on Step Stress-partially Accelerated Life Testing for Odd Generalized Nadrajah Haghighi Exponential Distribution

Sarah Mohammad Behairy*, Neama Taher Al-Sayed*

Department of Statistics, Faculty of Commerce, AL-Azhar University (Girls' Branch), Cairo, Egypt

Abstract

Accelerated or partially accelerated life tests are particularly significant in life testing experiments since they save time and cost. Partially accelerated life tests are carried out when the data from accelerated life testing cannot be extrapolated to usual conditions. The step stress-partially accelerated life test is proposed in this study based on a Type II censoring scheme and supposing that the lifetimes of units at usual conditions follow the odd generalized Nadarajah Haghighi exponential distribution. For the unknown parameters and acceleration factor, the maximum likelihood estimators are obtained. The balanced square error loss function as a symmetric loss function and the balanced linear exponential loss function as an asymmetric loss function are used to derive Bayes estimators based on informative priors. Finally, the performance of the proposed maximum likelihood and Bayes estimates is evaluated through a simulation study and an application using real data sets.

Keywords: Asymptotic information matrix, Balanced loss function, Censored samples, Odd generalized Nadarajah Haghighi exponential distribution

1. Introduction

Manufacturers are being supported to design and produce highly reliable products as market competition and customer expectations increase. The time to market is getting shorter and shorter, so it is necessary to evaluate and estimate a product's reliability through the design and development stage. Additionally, manufacturing designs continue to develop as a result of technological development, which makes it harder and harder to find out how long products or materials with high reliability will last when examined according to usual conditions. Therefore, in the manufacturing industry, *accelerated life testing* (ALT) or *partially accelerated life testing* (PALT) is preferred to get sufficient failure data rapidly and to examine its relationship with external stress variables. ALT or PALT provides information quickly on the life distribution of the materials or products by testing them at higher than usual levels of stress such as high temperature, voltage, pressure, vibration or load to induce early failures. A lot of time, manpower, resources, and money might be saved by using this testing.

The fundamental principle in ALT is that a life-stress connection exists or may be presumed, so that the data collected from accelerated conditions can be extended to usual conditions. PALT is usually applied in situations when such a relationship cannot be known or supposed.

Therefore, in these situations, PALT is a more appropriate test to run, where tested units are put through either usual or accelerated conditions. The concept of ALT was introduced and researched by [1,2]. Many approaches can be used to apply stress,

Received 10 June 2023; revised 27 August 2023; accepted 1 September 2023.
Available online 27 October 2023

* Corresponding authors. Department of Statistics, Faculty of Commerce, AL-Azhar University (Girls' Branch), Cairo, Egypt.
E-mail addresses: comm.29801010128504@azhar.edu.eg (S.M. Behairy), neamaalsayed2452.el@azhar.edu.eg (N.T. Al-Sayed).

https://doi.org/10.58675/2636-3305.1652

including constant stress, progressive stress, step stress, and among others.

A specimen is put through to increase levels of stress in step stress-ALT loading. Initially, a specimen is put under to a constant stress for a predetermined period of time. If it does not fail, it is put through to a higher level of stress for a set period of time. Thus, the stress applied to a specimen is gradually increased until it fails. Typically, all specimens proceed with the same stress level and test time patterning. The advantage of step stress is the reduction of the test duration, because if failure data collected are not enough in one stress, a higher stress could be set to increase the probability of failure, i.e. it can further reduce test time and variability of the failure times. The following publications provide further information about step stress-ALT, see [3−9] among others.

In a *step stress-PALT* (SS-PALT) a test unit is initially performed under usual conditions and if it does not fail after a certain period of time or number of failures, it is then run under accelerated conditions until failure happens or the observation is censored. The aim of this study is to obtain more failure data in a limited period of time without necessarily putting all test units to high-stress levels. Several studies have been conducted using Bayesian and non-Bayesian estimation based on SS-PALT under Type II censoring, see [10−14].

The *odd generalized Nadarajah Haghighi exponential* (OGNH-E) distribution as special case of the odd generalized Nadarajah Haghighi generated family distributions with application to exponential model was proposed by [15] its *cumulative distribution function* (cdf) is given by

$$F(x;\underline{\vartheta}) = 1 - exp\{1 - [\varphi(\theta\lambda)]^{\alpha}\}, x > 0, (\underline{\vartheta} > \underline{0}), \tag{1}$$

where

$$\varphi(\theta\lambda) = 1 + \left(e^{\theta x} - 1\right)^{\lambda}, \underline{\vartheta} = (\theta, \lambda, \alpha)', \tag{2}$$

the *probability density function* (pdf) corresponding to (1) is given by

$$f(x;\underline{\vartheta}) = \alpha\lambda\theta e^{\theta\lambda x}\left(1 - e^{-\theta x}\right)^{(\lambda-1)}[\varphi(\theta\lambda)]^{\alpha-1} exp[1 - (\varphi(\theta\lambda))^{\alpha}],$$

$$x > 0, (\underline{\vartheta} > \underline{0}), \tag{3}$$

where θ, λ and α are shape parameters.

The *reliability function* (rf) and *hazard rate function* (hrf) are given, respectively, by:

$$R(x;\underline{\vartheta}) = exp\{1 - [\varphi(\theta\lambda)]^{\alpha}\}, x > 0, (\underline{\vartheta} > \underline{0}), \tag{4}$$

and

$$h(x;\underline{\vartheta}) = \alpha\lambda\theta e^{\theta\lambda x}\left(1 - e^{-\theta x}\right)^{(\lambda-1)}[\varphi(\theta\lambda)]^{\alpha-1}, \quad x > 0, (\underline{\vartheta} > \underline{0}). \tag{5}$$

Figures 1 and 2.

One can see that the plots of the hrf of the OGNH-E distribution is a bathtub shape for different value of parameters, so the OGNH-E distribution is a flexible reliability model and it is suitable for studying PALT model. Some statistical properties were attained by [15]. Additionally, they derived the *maximum likelihood* (ML) estimators, asymptotic variances and covariance matrix of the ML estimators and CIs for the parameters.

This paper is structured as: in Section 2, the basic assumptions are given. The ML estimators for the parameters, the acceleration factor and *confidence intervals* (CIs) are obtained in Section 3. In Section 4, Bayesian point estimation and Bayesian *credible intervals* (BCIs) for the unknown parameters and the acceleration factor for SS-PALT based on Type II censored data under the *balanced square error loss* (BSEL) function and *balanced linear exponential loss* (BLL) function are discussed. A simulation study and an application using two real data sets are given to illustrate the theoretical results in section 5. Finally, some general conclusions are presented in section 6.

2. The Basic Assumptions

The basic assumptions are given in this section.

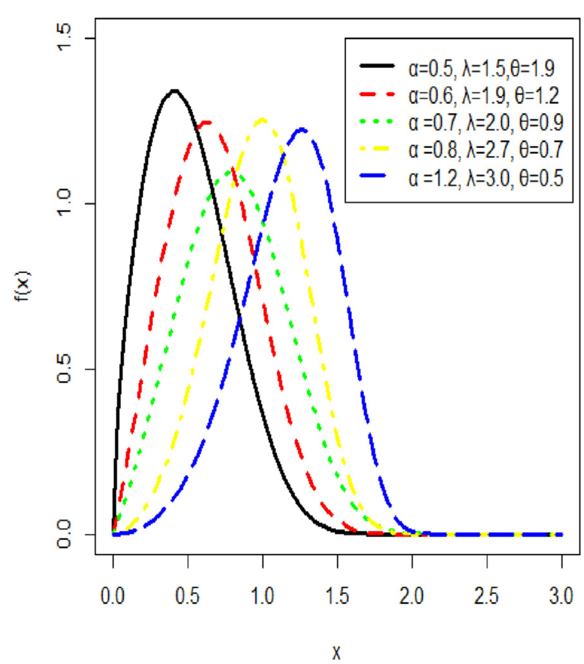

Fig. 1. Different shapes for the probability density function.

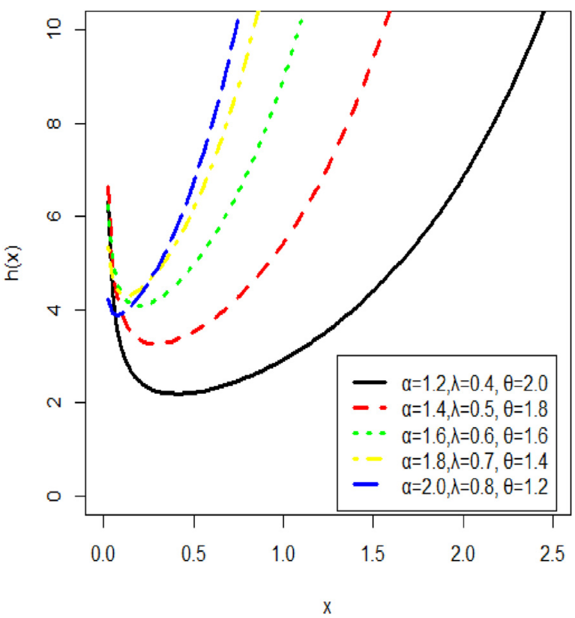

Fig. 2. Different shapes for the hazard rate function.

(1) The OGNH-E distribution is used for determining X is the lifetime of a unit under usual conditions.
(2) The failure times Y_j ; $j = 1, 2, \ldots, n_i$ are *independent and identically distributed* (i.i.d) random variables.
(3) z_1 and z_2 (usual and high) are the two stress levels that are employed.
(4) The total lifetime of test items designated by Y passes through two parts, which are usual and accelerated conditions. Then, the lifetime of a unit under SS-PALT is defined as

$$Y = \begin{cases} X \text{ if } X \le y_{n_1} \\ y_{n_1} + \beta^{-1}\left(X - y_{n_1}\right) \text{ if } X > y_{n_1} \end{cases}, \tag{6}$$

where X denotes the lifetime under the stress level z_1 and y_{n_1} is the lifetime of the failure unit order ${n_1}^{th}$, which is the latest failure under usual condition. The lifetime of a test item follows the OGNH-E distribution for any level of stress.

Test procedure.

(1) Assume that n test items are first set to a usual stress z_1 and run until time y_{n_1} when actually n_1 failures are occurred during testing at the stress level z_1. The test is ended if the number of failures attains the predetermined n_1 units, where $n_1 = \pi_1 n$ and π_1 is a percentage of the total test units used in the test at usual use that was predetermined at $0 < \pi_1 < 1$.
(2) The items are placed according to high-stress z_2 and tested until time y_{n_2} when exactly n_2 failures are observed for units that do not fail under the usual use condition $(n - n_1)$, where $n_2 = \pi_2 n$ and π_2 is a percentage of the total test units placed under test according to accelerated condition that predetermined $0 < \pi_2 < 1$ and $0 < \pi_1 + \pi_2 < 1$. After that, the remaining $n_c = n - n_1 - n_2$ items are then censored.

In this situation, if the item has not failed after a predetermined number of failures, the test condition is changed to an increased level of stress, and it is continued until another determined number of failures happens or the observation is censored. The impact of this change is to multiply the units of residual lifetime by reverse of an acceleration factor β, where $Y = \beta^{-1}X$, is the acceleration factor that is the ratio of mean life under usual conditions to mean life according to accelerated conditions and $\beta > 1$. As a result, the total lifetime of a test unit, designated by Y, passes through two steps, the first of which is the usual use condition and the second of which is the accelerated use condition, correspondingly. See, [11,13].

The pdf of an item of total lifetime Y under Type II censoring in a simple SS-PALT is provided by:

$$Y = \begin{cases} f_1(y; \underline{\vartheta}) \text{ if } y \le y_{n_1} \\ f_2(y; \underline{\Phi}) \text{ if } y > y_{n_1} \end{cases}, \tag{7}$$

where $f_1(y; \underline{\vartheta})$, follow an OGNH-E distribution with pdf

$$f_1(y; \underline{\vartheta}) = \alpha\lambda\theta e^{\theta\lambda y_j}\left(1 - e^{-\theta y_j}\right)^{(\lambda-1)}\left[\varphi_j(\theta\lambda)\right]^{\alpha-1} \times exp\left[1 - \left(\varphi_j(\theta\lambda)\right)^{\alpha}\right], y \le y_{n_1}, (\underline{\vartheta} > \underline{0}). \tag{8}$$

By using the transformation-variable technique $f_1(y; \underline{\vartheta})$ and the model shown in (3), the pdf of $f_2(y; \underline{\Phi})$, is obtained as below

$$f_2(y; \underline{\Phi}) = \alpha\lambda\theta e^{\theta\lambda \hbar_j}\left(1 - e^{-\theta \hbar_j}\right)^{(\lambda-1)}\left[\varphi_j(\theta\lambda\beta)\right]^{\alpha-1} \times exp\left[1 - \left(\varphi_j(\theta\lambda\beta)\right)^{\alpha}\right],$$

$$y > y_{n_1}, (\underline{\vartheta} > \underline{0});\ \beta > 1, \tag{9}$$

where

$$\begin{cases} \underline{\Phi} = (\theta, \lambda, \alpha, \beta)', \varphi_j(\theta\lambda) = 1 + \left(e^{\theta y_j} - 1\right)^{\lambda}, \\ \varphi_j(\theta\lambda\beta) = 1 + \left(e^{\theta \hbar_j} - 1\right)^{\lambda}, \hbar_j = \beta\left(y_j - y_{n1}\right) + y_{n1}, \end{cases} \tag{10}$$

for a unit tested at acceleration conditions the cdf, rf and hrf of pdf $f_2(y; \underline{\Phi})$ are provided by

$$F(y; \underline{\Phi}) = 1 - exp\{1 - [\varphi_r(\theta\lambda\beta)]^{\alpha}\}, y > y_{n_1}, (\underline{\vartheta} > \underline{0});\ \beta > 1, \tag{11}$$

$$R(y;\underline{\Phi}) = exp\{1-[\varphi_r(\theta\lambda\beta)]^{\alpha}\}, y > y_{n_1}, (\underline{\vartheta} > \underline{0});\ \beta > 1, \tag{12}$$

and

$$h(y;\underline{\Phi}) = \frac{\alpha\lambda\theta e^{\theta\lambda \mathscr{k}_j}(1-e^{-\theta \mathscr{k}_j})^{(\lambda-1)}[\varphi_j(\theta\lambda\beta)]^{\alpha-1} exp[1-(\varphi_j(\theta\lambda\beta))^{\alpha}]}{exp\{1-[\varphi_r(\theta\lambda\beta)]^{\alpha}\}}, y > y_{n_1}, (\underline{\vartheta} > \underline{0}); \beta > 1, \tag{13}$$

where

$$\varphi_r(\theta\lambda\beta) = 1 + \left(e^{\theta\left[\beta\left(y_r - y_{n1}\right)+y_{n1}\right]} - 1\right)^{\lambda}, \tag{14}$$

the observed values of the total life time Y can be obtained by

$$y_{(1)} \le y_{(2)} \le \ldots \le y_{(n_1)} \le y_{(n_1+1)} \le \ldots \le y_{(n_1+n_2-1)} \le y_r.$$

3. Classical Estimation

Point estimation of the unknown parameters and the acceleration factor for the OGNH-E distribution for SS-PALT using Type II censored data are investigated in this section. The interval estimation of the unknown parameters are also derived.

The *likelihood function* (LF) of the observations and n_c censored data according to SS-PALT based on Type II censoring is as below

$$L(\underline{\Phi}) \propto \prod_{j=1}^{n_1} \alpha\lambda\theta e^{\theta\lambda y_j}\left(1-e^{-\theta y_j}\right)^{(\lambda-1)}\left[\varphi_j(\theta\lambda)\right]^{\alpha-1} \times exp\left[1-\left(\varphi_j(\theta\lambda)\right)^{\alpha}\right]$$

$$\times \prod_{j=1}^{n_2} \alpha\lambda\theta\beta e^{\theta\lambda \mathscr{k}_j}\left(1-e^{-\theta \mathscr{k}_j}\right)^{(\lambda-1)}\left[\varphi_j(\theta\lambda\beta)\right]^{\alpha-1} \times\ exp\left[1-\left(\varphi_j(\theta\lambda\beta)\right)^{\alpha}\right]$$

$$\times \prod_{j=1}^{n_c} exp\{1-[\varphi_r(\theta\lambda\beta)]^{\alpha}\}, \tag{15}$$

where $\varphi_j(\theta\lambda), \varphi_j(\theta\lambda\beta), \mathscr{k}_j$ and $\varphi_r(\theta\lambda\beta)$ are provided by (10) and (14), consequently.

3.1. Point estimation

By maximizing the logarithm of (15), represented by l then the ML estimators of α, λ, θ and β are produced and having the form:

$$l = lnL(\underline{\Phi}) \propto n\pi_1\, ln(\alpha) + n\pi_1\, ln(\lambda) + n\pi_1\, ln(\theta) + \sum_{j=1}^{n_1}\theta\lambda y_j + (\lambda-1) \times \sum_{j=1}^{n_1} ln\left(1-e^{-\theta y_j}\right) + (\alpha-1)\sum_{j=1}^{n_1} ln[\varphi_j(\theta\lambda)] + \sum_{j=1}^{n_1}\left[1-\left(\varphi_j(\theta\lambda)\right)^{\alpha}\right]$$

$$+ n\pi_2\, ln(\alpha) + n\pi_2\, ln(\lambda) + n\pi_2\, ln(\theta) + n\pi_2\, ln(\beta) + \sum_{j=1}^{n_2}\theta\lambda \mathscr{k}_j + (\lambda-1) \times \sum_{j=1}^{n_2} ln\left(1-e^{-\theta \mathscr{k}_j}\right) + (\alpha-1)\sum_{j=1}^{n_2} ln[\varphi_j(\theta\lambda\beta)] + \sum_{j=1}^{n_2}\left[1-\left(\varphi_j(\theta\lambda\beta)\right)^{\alpha}\right]$$

$$+ n_c\{1-[\varphi_r(\theta\lambda\beta)]^{\alpha}\}. \tag{16}$$

The partial derivatives of the logarithm for the LF with regard to α, λ, θ and β are presented below:

$$\frac{\partial l}{\partial \alpha} = \frac{n(\pi_1+\pi_2)}{\alpha} + \sum_{j=1}^{n_1} ln\left(1-e^{-\theta y_j}\right) - \sum_{j=1}^{n_1}\left(\varphi_j(\theta\lambda)\right)^{\alpha} ln\left[\varphi_j(\theta\lambda)\right] + \sum_{j=1}^{n_2} ln\left[\varphi_j(\theta\lambda\beta)\right]$$

$$- \sum_{j=1}^{n_1}\left(\varphi_j(\theta\lambda\beta)\right)^{\alpha} ln\left[\varphi_j(\theta\lambda\beta)\right] - n_c\{[\varphi_r(\theta\lambda\beta)]^{\alpha} ln[\varphi_r(\theta\lambda\beta)]\}, \tag{17}$$

$$\frac{\partial l}{\partial \lambda}=\frac{n(\pi_1+\pi_2)}{\lambda}+\theta\sum_{j=1}^{n_1}y_j+\sum_{j=1}^{n_1}ln\left(1-e^{-\theta y_j}\right)$$
$$+(\alpha-1)\sum_{j=1}^{n_1}\frac{\left(e^{\theta y_j}-1\right)^{\lambda}ln\left(e^{\theta y_j}-1\right)}{\left[\varphi_j(\theta\lambda)\right]}$$
$$-\alpha\sum_{j=1}^{n_1}\left(\varphi_j(\theta\lambda)\right)^{\alpha-1}\left(e^{\theta y_j}-1\right)^{\lambda}ln\left(e^{\theta y_j}-1\right)+\theta\sum_{j=1}^{n_2}\kappa_j$$
$$+\sum_{j=1}^{n_2}ln\left(1-e^{-\theta\kappa_j}\right)$$
$$+(\alpha-1)\sum_{j=1}^{n_1}\frac{\left(e^{\theta\kappa_j}-1\right)^{\lambda}ln\left(e^{\theta\kappa_j}-1\right)}{\left[\varphi_j(\theta\lambda\beta)\right]}-\alpha\sum_{j=1}^{n_1}\left(\varphi_j(\theta\lambda\beta)\right)^{\alpha-1}$$
$$\times\left(e^{\theta\kappa_j}-1\right)^{\lambda}ln\left(e^{\theta\kappa_j}-1\right)-\alpha n_c\left\{\left[\varphi_r(\theta\lambda\beta)\right]^{\alpha-1}\right.$$
$$\left.\times\left(e^{\theta\left[\beta\left(y_r-y_{n1}\right)+y_{n1}\right]}-1\right)^{\lambda}ln\left(e^{\theta\left[\beta\left(y_r-y_{n1}\right)+y_{n1}\right]}-1\right)\right\}, \tag{18}$$

$$\frac{\partial l}{\partial \theta}=\frac{n(\pi_1+\pi_2)}{\theta}+\lambda\sum_{j=1}^{n_1}y_j+(\lambda-1)\sum_{j=1}^{n_1}\frac{y_je^{\theta y_j}}{\left(e^{\theta y_j}-1\right)}$$
$$+\lambda(\alpha-1)\sum_{j=1}^{n_1}\frac{y_je^{\theta y_j}\left(e^{\theta y_j}-1\right)^{\lambda-1}}{\left[\varphi_j(\theta\lambda)\right]}$$
$$-\alpha\lambda\sum_{j=1}^{n_1}\left(\varphi_j(\theta\lambda)\right)^{\alpha-1}y_je^{\theta y_j}\left(e^{\theta y_j}-1\right)^{\lambda-1}+\lambda\sum_{j=1}^{n_2}\kappa_j$$
$$+(\lambda-1)\sum_{j=1}^{n_2}\frac{\kappa_je^{\theta\kappa_j}}{\left(e^{\theta\kappa_j}-1\right)}$$
$$+\lambda(\alpha-1)\sum_{j=1}^{n_2}\frac{y_je^{\theta\kappa_j}\left(e^{\theta\kappa_j}-1\right)^{\lambda-1}}{\left[\varphi_j(\theta\lambda\beta)\right]}$$
$$-\alpha\lambda\sum_{j=1}^{n_2}\left(\varphi_j(\theta\lambda\beta)\right)^{\alpha-1}\kappa_je^{\theta\kappa_j}\left(e^{\theta\kappa_j}-1\right)^{\lambda-1}$$
$$-\alpha\lambda n_c\left\{\left[\varphi_r(\theta\lambda\beta)\right]^{\alpha-1}\left(e^{\theta\left[\beta\left(y_r-y_{n1}\right)+y_{n1}\right]}-1\right)^{\lambda-1}\right.$$
$$\left.e^{\theta\left[\beta\left(y_r-y_{n1}\right)+y_{n1}\right]}\left[\left(y_r-y_{n1}\right)+y_{n1}\right]\right\}, \tag{19}$$

and

$$\frac{\partial l}{\partial \beta}=\frac{n\pi_2}{\beta}+\lambda\theta\sum_{j=1}^{n_2}\left(y_j-y_{n1}\right)+\theta(\lambda-1)\sum_{j=1}^{n_2}\frac{\left(y_j-y_{n1}\right)e^{-\theta\kappa_j}}{\left(1-e^{-\theta\kappa_j}\right)}$$
$$+\theta\lambda(\alpha-1)\sum_{j=1}^{n_2}\frac{\left(e^{\theta\kappa_j}-1\right)^{\lambda-1}}{\left[\varphi_j(\theta\lambda\beta)\right]}$$
$$\times\sum_{j=1}^{n_2}\frac{\left(e^{\theta\kappa_j}-1\right)^{\lambda-1}\left(y_j-y_{n1}\right)e^{\theta\kappa_j}}{\left[\varphi_j(\theta\lambda\beta)\right]}$$
$$-\alpha\lambda\theta\sum_{j=1}^{n_2}\left(\varphi_j(\theta\lambda\beta)\right)^{\alpha-1}\left(e^{\theta\kappa_j}-1\right)^{\lambda-1}\left(y_j-y_{n1}\right)e^{\theta\kappa_j}$$
$$-\alpha\lambda\theta n_c\left\{\left[\varphi_r(\theta\lambda\beta)\right]^{\alpha-1}\left(e^{\theta\left[\beta\left(y_r-y_{n1}\right)+y_{n1}\right]}-1\right)^{\lambda-1}\right.$$
$$\left.e^{\theta\left[\beta\left(y_r-y_{n1}\right)+y_{n1}\right]}\left[\left(y_r-y_{n1}\right)\right]\right\}, \tag{20}$$

where $\varphi_j(\theta\lambda), \varphi_j(\theta\lambda\beta), \kappa_j$ and $\varphi_r(\theta\lambda\beta)$ are provided by (10) and (14), consequently.

Equations (17)−(20) are set to zero to obtain the ML estimators. ML estimations for the parameters α, λ, θ and β can be derived by numerically solving the system of non-linear equations employing the Newton−Raphson technique.

3.2. Asymptotic confidence intervals

The partial second derivatives of the logarithm of the LF are used to construct the *asymptotic variance covariance matrix* for the estimators α, λ, θ and β dependent to the inverse *asymptotic Fisher information matrix* (AFIM).

The AFIM can be written as follows:

$$\tilde{I}=-\left[\frac{\partial^2 l}{\partial\Phi_i\partial\Phi_j}\right], \text{where } i,j=1,2,3,4, \tag{21}$$

where $\Phi_1=\alpha, \Phi_2=\lambda, \Phi_3=\theta$ and $\Phi_4=\beta$.

According to regularity criteria, the ML estimators are consistent, asymptotically unbiased and asymptotically normally distributed in large sample sizes. As a result, the *asymptotic CIs* (ACI) of the parameters may be determined by $\mathrm{P}\left[-\mathrm{Z}<\frac{\widehat{\Phi}_\mathrm{i}-\Phi_\mathrm{i}}{\sqrt{\widehat{\mathrm{var}}(\widehat{\Phi}_\mathrm{i})}}<\mathrm{Z}\right]=1-\tau$ where Z is the $100\left(1-\frac{\tau}{2}\right)$ th standard normal percentile.

The two-sided approximate $100(1-\tau)\%$ the CIs are

$$LL_{\Phi_i}=\widehat{\Phi}_i-Z_{\frac{\tau}{2}}\sqrt{\widehat{\mathrm{var}}(\widehat{\Phi}_i)}\text{ and }UL_{\Phi_i}=\widehat{\Phi}_i+Z_{\frac{\tau}{2}}\sqrt{\widehat{\mathrm{var}}(\widehat{\Phi}_i)}, \tag{22}$$

where LL_{Φ_i} and UL_{Φ_i} are the *lower limit* (LL) and *upper limit* (UL), $\widehat{\mathrm{var}}(\widehat{\Phi}_i)$ is the i^{th} diagonal element of asymptotic variance covariance matrix and $\widehat{\Phi}_i$ *is* $\widehat{\alpha},\widehat{\lambda},\widehat{\theta}$ or $\widehat{\beta}$, respectively.

4. Bayesian Estimation

The parameters and acceleration factor of the OGNH-E distribution for the SS-PALT are estimated using the Bayesian point and BCIs based on Type II censored samples in this section. The BSEL and BLL loss function are employed to demonstrate Bayesian estimator for the parameters and acceleration factor of the OGNH-E distribution.

When the sample size is too small for more advanced statistical analysis to be conducted, Bayesian statistics can be a useful tool for solving specific inference problems. The Bayes estimator is an estimator that reduces the posterior expected value of a loss function, or the posterior expected loss. Alternatively, it maximizes the posterior expectation of a useful function.

The unknown parameters are regarded as random variables in Bayesian analysis. The use of prior information about the unknown parameters is concerned. The posterior information is acquired to estimate the behavior of the items assuming usual conditions. Because the integrations in these situations are complex so numerical results are produced.

Prior distributions for the parameters are considered to be the informative priors. Assuming that the parameters vector $\underline{\Phi}=(\underline{\vartheta},\beta)$ are independent and belongs to gamma distribution, where $\Phi_i\sim$ gamma (a_i,b_i) and a_i,b_i are the hyper-parameters of the prior distribution for $i=1,\dots,4$. Then, for all the unknown parameters, the joint prior distribution has a joint pdf provided by

$$\pi(\underline{\Phi})\propto\prod_{i=1}^{4}{\Phi_i}^{a_i-1}\exp(-b_i\Phi_i),\ \beta>1;\ (\underline{\vartheta},a_i,b_i>\underline{0}), \tag{23}$$

where $\Phi_1=\alpha,\Phi_2=\lambda,\Phi_3=\theta$, $\Phi_4=\beta$ and $\underline{\vartheta}=(\alpha,\lambda,\theta)'$.

The joint posterior distribution for $\underline{\Phi}$ can be determined by combining the LF in (15) and the joint prior distribution given by (23) as follows

$$\begin{aligned}\pi\left(\underline{\Phi}\middle|\,\underline{y}\right)&=A\alpha^{n(\pi_1+\pi_2)+a_1-1}\lambda^{n(\pi_1+\pi_2)+a_2-1}\theta^{n(\pi_1+\pi_2)+a_3-1}\beta^{n\pi_2+a_4-1}\\&\times\exp[-(b_1\alpha+b_2\lambda+b_3\theta+b_4\beta)]\\&\times\prod_{j=1}^{n_1}e^{\theta\lambda y_j}\left(1-e^{-\theta y_j}\right)^{(\lambda-1)}\left[\varphi_j(\theta\lambda)\right]^{\alpha-1}\\&\times exp\left[1-\left(\varphi_j(\theta\lambda)\right)^{\alpha}\right]\prod_{j=1}^{n_2}e^{\theta\lambda\mathscr{k}_j}\left(1-e^{-\theta\mathscr{k}_j}\right)^{(\lambda-1)}\left[\varphi_j(\theta\lambda\beta)\right]^{\alpha-1}\\&\times exp\left[1-\left(\varphi_j(\theta\lambda\beta)\right)^{\alpha}\right]\prod_{j=1}^{n_c}exp\{1-[\varphi_r(\theta\lambda\beta)]^{\alpha}\},\end{aligned} \tag{24}$$

where $\varphi_j(\theta\lambda),\varphi_j(\theta\lambda\beta),\mathscr{k}_j$ and $\varphi_r(\theta\lambda\beta)$ are presented by (10) and (14), respectively, a_i,b_i are the hyper-parameters of the prior distribution for $i=1,\dots,4$, A is the normalizing constant and can be derived as follows

$$\int_{\underline{\Phi}}\pi\left(\underline{\Phi}\middle|\,\underline{y}\right)d\underline{\Phi}=1, \tag{25}$$

where

$$\int_{\underline{\Phi}}=\int_{\alpha}\int_{\lambda}\int_{\theta}\int_{\beta}\text{ and }d\underline{\Phi}=d\alpha\,d\lambda\,d\theta d\beta. \tag{26}$$

4.1. Bayesian estimation based on balanced loss functions

Asymmetric and symmetric loss function are the two categories into which loss function are divided. There are many different forms of symmetric and asymmetric loss function.

[16] introduced the class of the *balanced loss function* (BLF). An extended class of the BLF was proposed by [17], with the following form:

$$L^*(\theta,\tilde{\theta})=\omega\, l(\theta,\widehat{\theta})+(1-\omega)\, l(\theta,\tilde{\theta}), \tag{27}$$

where $l(\theta,\tilde{\theta})$ represents any loss function, $\widehat{\theta}$ is a selected target estimator of θ and the weight $\omega\epsilon[0,1]$. The BLF employs in a variety of loss functions for example the absolute error loss, *squared error loss* (SEL), entropy and *linear exponential* (LINEX) functions. Based on the BLF, the estimator of a function is a mixture of the Bayes estimator under any loss function and the ML estimator, least squares estimators or any other estimator.

The Bayes estimator of θ, using the BSEL function is as follows:

$$\tilde{\theta}_{\mathrm{BSE}} = \omega\, \widehat{\theta}_{\mathrm{ML}} + (1-\omega)\tilde{\theta}_{\mathrm{SE}}, \tag{28}$$

where $\widehat{\theta}_{\mathrm{ML}}$ is the ML estimator of θ and $\tilde{\theta}_{\mathrm{SE}}$ is its Bayes estimator under SEL function. Also, the Bayes estimator based on the BLL function of θ is derived as:

$$\tilde{\theta}_{BLL} = \frac{-1}{v} \ln\left\{\omega\, exp(-v\widehat{\theta}_{ML}) + (1-\omega)\, E\left(exp(-v\theta)\middle|\underline{x}\right)\right\}, \tag{29}$$

where $v \neq 0$ is the shape parameter for the BLL function.

Other estimators, such as the least squares estimator may be employed instead of the ML estimator. Several studies used the symmetric and asymmetric BLF to construct Bayes estimators for several other distributions, see [18–20].

4.1.1. Bayes estimators under balanced squared error loss function

From (24) and (28), the Bayes estimators for the parameters using the BSEL function can be obtained as shown below

$$\tilde{\Phi}_{iBSEL} = \omega\widehat{\Phi}_{iML} + (1-\omega)\int_{\underline{\Phi}} \Phi_i \pi\left(\underline{\Phi}\middle|\underline{y}\right) d\underline{\Phi}, \tag{30}$$

where $\pi\left(\underline{\Phi}\middle|\, \underline{y}\right)$ is given by (24) and $\int_{\underline{\Phi}}, d\underline{\Phi}$, are given by (26).

By replacing Φ_i by α, λ, θ and β in (30), one can obtain the Bayes estimators of the parameters under BSEL function.

4.1.2. Bayes estimators under balanced linear exponential loss function

From (24) and (29) the Bayes estimators of the parameters under BLL function can be obtained as follows

$$\tilde{\Phi}_{i\mathrm{BLL}} = \frac{-1}{v}\ln\left\{\omega \exp(-v\widehat{\Phi}_{iML}) + (1-\omega)\int_{\underline{\Phi}} \exp(-\Phi_i v)\pi\left(\underline{\Phi}\middle|\underline{y}\right) d\underline{\Phi},\right. \tag{31}$$

when Φ_i replaces by α, λ, θ and β in (31), one can obtain the Bayes estimators of the parameters under BLL function.

4.2. Bayesian credible intervals

In general $[\mathrm{L}(\underline{y}) < \Phi_i < \mathrm{U}(\underline{y})]$ is a 100(1-$\boldsymbol{\tau}$) % BCIs for Φ_i where $\Phi_i = \alpha, \lambda, \theta$ or β if

$$P\left(\left(L\left(\underline{y}\right) < \Phi_i < U\left(\underline{y}\right)\middle|\underline{y}\right) = \int_{\mathrm{L}\left(\underline{y}\right)}^{\mathrm{U}\left(\underline{y}\right)} \pi\left(\Phi_i\middle|\underline{y}\right) d\Phi_i = 1-\tau. \tag{32}$$

The LL and UL $\left[L\left(\underline{y}\right), U\left(\underline{y}\right)\right]$ can be obtained by evaluating

$$P\left(\Phi_i > L\left(\underline{y}\right)\middle|\underline{y}\right) = 1-\frac{\tau}{2} \text{ and } P\left(\Phi_i > U\left(\underline{y}\right)\middle|\underline{y}\right) = \frac{\tau}{2}, \tag{33}$$

where $\Phi_i = \alpha, \lambda, \theta$ or β, $L\left(\underline{y}\right), U\left(\underline{y}\right)$ are the LL and UL of BCIs.

5. Numerical Illustration

In this section, the accuracy of theoretical results of ML and Bayes estimates are investigated using simulated and real data sets.

5.1. Simulation algorithm

A simulation study is carried out in this subsection to demonstrate the efficiency of the provided ML and Bayes estimates for SS-PALT using Type II censored data generated from the OGNH-E $(\alpha, \lambda, \theta)$ distribution. In ML and Bayes estimates (point and interval) are calculated. For all simulation investigations, the Mathematica 9 and R programming language are used.

5.1.1. For maximum likelihood method using Mathematica 9

The following are the steps of the simulation process using Type II censored data:

Step 1: Random samples of size n are generated from the OGNH-E $(\alpha, \lambda, \theta)$ distribution for specified values of $\underline{\vartheta}$.

[15] provide the following transformation between the uniform distribution and the OGNH-E distribution

$$x_u = \frac{1}{\theta} ln\left[1 + \left(1 - ln(1-u)^{\frac{1}{\alpha}} - 1\right)^{\frac{1}{\lambda}}\right], 0 < u < 1.$$

Where $u_1, u_2, \ldots, u_n$ are random sample from uniform (0,1).

Step 2: Our experiment is conducted using Type II censoring, which means it ends when the first failure occurs. Each of the n test items is initially

Table 1. The goodness of fit measures for fitted models of Application 1.

Model	KS	AD	CM	LL	AIC	BIC	CIAC	HQIC
OGNH-E	0.139	0.871	0.129	101.52	209.05	214.79	209.57	211.24
GLE	0.122	0.885	0.131	101.77	209.54	215.28	210.07	211.73
APTW	0.140	0.892	0.133	102.12	210.64	216.38	211.16	212.83
Ex-GLE	0.115	0.896	0.139	102.28	210.57	217.22	211.46	213.48
EL	0.142	0.975	0.155	102.96	211.93	217.66	212.45	214.11
APTE	0.220	1.027	0.165	107.09	218.19	222.01	218.45	219.65

Table 2. The goodness of fit measures for fitted models of Application 2.

Model	KS	AD	CM	LL	AIC	BIC	CIAC	HQIC
OGNH-E	0.063	0.473	0.078	168.64	343.28	351.62	343.49	346.67
GLE	0.092	0.516	0.802	168.70	345.17	353.51	345.38	348.56
APTW	0.144	0.573	0.087	170.60	347.21	355.55	347.42	350.60
Ex-GLE	0.087	0.565	0.084	168.84	345.68	354.79	346.03	350.19
EL	0.093	0.591	0.149	170.82	347.65	355.99	347.86	351.03
APTE	0.310	2.041	0.334	245.59	495.18	500.74	495.28	497.44

performed under usual use condition, if the number of failures exceeds $n_1 = \pi_1 n$ items, which is predetermined, the test is stopped, where $\pi_1 = 20\%$ is a percentage of total test items tested under usual use condition. The remaining items (80% n) are then placed on accelerated use condition and run until the number of failures exceeds n_2 products, at which point the test is ended, where $n_2 = \pi_2 n$ and $\pi_2 = 20\%$ is a percentage of all test items put on test according to the predetermined accelerated condition.

Step 3: Under Type II censored samples, the distribution parameters and acceleration factor are estimated in SS-PALT for each sample and for each set of parameters. To get the estimates of α, λ, θ, and β, the Newton Raphson approach is used to solve the nonlinear Equations (17)−(20).

Repeat all of the preceding steps N times, where N denotes a predetermined number of simulated samples and N = 1000 is the number of repetitions.

Step 4: Some accuracy metrics are used to evaluate the performance of the estimates. It is convenient to employ average and *estimated risk* (ER) to analyze the precision and volatility of the estimations, where $\overline{\widehat{\Phi}_i} = \frac{\sum_{j=1}^{N} \widehat{\Phi}_i^j}{N}$, $\Phi_i = \alpha, \lambda, \theta$ and β and ER $= \frac{\sum_{j=1}^{N} (estimate-true\ value)^2}{N}$.

Step 5: Equation (22) is used to get the two-sided CIs with confidence levels for the acceleration factor and the two parameters.

Tables 3 and 4 show the simulation results of the ML estimates for samples of size (n = 30, 60, 100). The values of the parameters for each sample size are chosen as (case 1, $\alpha = 0.7,\ \lambda = 0.7,\ \theta = 0.6,\ \beta = 1.1,\ \pi_1 = 30\%$ and $\pi_2 = 60\%$) and (case 2, $\alpha = 0.7,\ \lambda = 0.7,\ \theta = 0.6,\ \beta = 2.1, \pi_1 = 30\%\ and\ \pi_2 = 60\%$).

5.1.2. For Bayesian estimate using R programing

Step 1: Using the preceding generation processes a Type II censored sample may be constructed from the OGNH-E distribution.

(1) The set of hyper parameters ($a_i = 10, 0.5, 0.1, 0.5,\ b_i = 10, 10, 20, 15$) for $i = 1, \ldots, 4$ consequently.

Table 3. Maximum likelihood averages, estimated risks and 95% CIs for the parameters $\alpha, \lambda, \theta, \beta$ based on Type II censoring ($N = 1000$, $\pi_1 = 30\%$, $\pi_2 = 60\%$) (Case 1, $\alpha = 0.7, \lambda = 0.7, \theta = 0.6, \beta = 1.1$).

n	π	Parameters	Averages	ERs	LL	UL	Length
		α	0.7362	0.1531	0.1249	1.3475	1.2226
30	$\pi_1 = 30\%$	λ	0.8149	0.1184	0.5435	1.0864	0.5429
	$\pi_2 = 60\%$	θ	0.3543	0.0426	0.0679	0.6408	0.5729
		β	0.9847	0.4657	0.0000	2.3030	2.3030
		α	0.8037	0.0924	0.7775	0.8299	0.0523
60	$\pi_1 = 30\%$	λ	0.8136	0.0987	0.7746	0.8526	0.0779
	$\pi_2 = 60\%$	θ	0.3497	0.0241	0.2711	0.4283	0.1571
		β	0.9311	0.0721	0.5219	1.3404	0.8186
		α	0.7262	0.0512	0.7181	0.7343	0.0163
100	$\pi_1 = 30\%$	λ	0.7468	0.0609	0.7415	0.7519	0.0105
	$\pi_2 = 60\%$	θ	0.5434	0.0023	0.5047	0.5821	0.0774
		β	1.0763	0.0011	1.0308	1.1217	0.0909

Table 4. Maximum likelihood averages, estimated risks and 95% CIs of the parameters $\alpha, \lambda, \theta, \beta$ based on Type II censoring (N = 1000, $\pi_1 = 30\%$, $\pi_2 = 60\%$) (Case 2, $\alpha = 0.7, \lambda = 0.7, \theta = 0.6, \beta = 2.1$).

n	π	Parameters	Averages	ERs	LL	UL	Length
		α	0.7685	0.3064	0.1803	1.7173	1.8976
30	$\pi_1 = 30\%$	λ	0.8269	0.1308	0.5242	1.1297	0.6055
		θ	0.5781	0.1056	0.0000	1.1965	1.1965
	$\pi_2 = 60\%$	β	1.5621	5.7703	0.0000	6.1507	6.1507
		α	0.8743	0.1439	0.7533	0.9953	0.2419
60	$\pi_1 = 30\%$	λ	0.8034	0.0924	0.7656	0.8412	0.0756
		θ	0.6825	0.0816	0.2519	1.1130	0.8610
	$\pi_2 = 60\%$	β	0.7209	4.4116	0.0000	3.8259	3.8259
		α	0.7262	0.0512	0.7159	0.7365	0.0206
100	$\pi_1 = 30\%$	λ	0.7509	0.0629	0.7397	0.7621	0.0224
		θ	0.4449	0.0034	0.4069	0.4829	0.0759
	$\pi_2 = 60\%$	β	2.1455	0.0116	1.9538	2.3373	0.3835

Table 5. Bayes averages, ERs and 95% Bayesian credible intervals for the parameters $\alpha, \lambda, \theta, \beta$ using informative prior under balanced square error loss function based on Type II censoring (Case 1, N = 5000, $\omega = 0.2, \alpha = 1.5, \lambda = 0.9, \theta = 0.8, \beta = 2.1, \pi_1 = 20\%, \pi_2 = 20\%, \pi_1 = 20\%, \pi_2 = 40\%$ and $\pi_1 = 20\%, \pi_2 = 70\%$).

n	π	Parameters	Averages	ERs	LL	UL	Length
		α	1.4968	0.0325	1.4628	1.5748	0.1121
20	$\pi_1 = 20\%$	λ	0.9590	0.0222	0.8725	0.9964	0.1239
	$\pi_2 = 20\%$	θ	0.8934	0.1265	0.8038	0.9608	0.1569
		β	1.8984	0.0159	2.0459	2.0959	0.0501
		α	1.7981	0.1714	1.5042	2.0520	0.5478
	$\pi_1 = 20\%$	λ	0.6543	0.1449	0.2910	0.8024	0.5114
	$\pi_2 = 40\%$	θ	0.4260	0.2971	0.0000	0.7658	0.9784
		β	1.5375	0.3632	0.8230	2.1918	1.3688
		α	1.1591	0.1997	0.4038	1.4491	1.0453
	$\pi_1 = 20\%$	λ	1.4501	0.4204	0.8194	1.9389	1.1195
	$\pi_2 = 70\%$	θ	1.2671	0.3496	0.8173	1.7369	0.9196
		β	1.3079	1.0137	0.4437	2.1053	1.6616
		α	1.4829	0.0321	1.4687	1.5233	0.0546
60	$\pi_1 = 20\%$	λ	0.9568	0.0009	0.8778	0.9692	0.0914
	$\pi_2 = 20\%$	θ	0.8635	0.0035	0.7985	0.8980	0.0996
		β	1.9197	0.0151	2.0979	2.1005	0.0026
		α	1.6209	0.0782	1.2357	1.9933	0.7576
	$\pi_1 = 20\%$	λ	0.9447	0.0180	0.6204	1.1223	0.5019
	$\pi_2 = 40\%$	θ	0.9153	0.0298	0.6801	1.1613	0.4812
		β	1.8021	0.0416	1.5389	2.1582	0.6193
		α	1.5015	0.0395	1.1483	1.8203	0.6719
	$\pi_1 = 20\%$	λ	0.7390	0.0769	0.0598	0.9569	0.8971
	$\pi_2 = 70\%$	θ	0.5976	0.1146	0.2407	0.7569	0.5163
		β	1.9532	0.4299	1.5084	2.5633	1.0549
		α	1.4779	1.5242E-5	1.4915	1.5036	0.0121
	$\pi_1 = 20\%$	λ	0.9357	3.1685E-5	0.8905	0.8967	0.0062
	$\pi_2 = 20\%$	θ	0.8168	4.6297E-5	0.7859	0.8026	0.0166
		β	1.9129	9.6342E-5	2.0834	2.0988	0.0153
		α	1.4734	0.0008	1.4487	1.5424	0.0937
100	$\pi_1 = 20\%$	λ	1.0089	0.0087	0.8892	1.0259	0.1367
	$\pi_2 = 40\%$	θ	0.8176	0.0008	0.7316	0.8399	0.1082
		β	1.9560	0.0026	2.0944	2.1831	0.0887
		α	1.4501	0.0026	1.3652	1.5144	0.1491
	$\pi_1 = 20\%$	λ	0.8586	0.0126	0.6829	0.8624	0.1795
	$\pi_2 = 70\%$	θ	0.7942	0.0045	0.6202	0.8350	0.2148
		β	1.8338	0.0146	1.8642	2.0684	0.2042

(2) In an informative prior, the gamma distribution is employed.

Step 2: From (30) and (31), simulation results of the Bayesian estimates of the parameters and acceleration factor using BSEL and BLL functions are obtained, and Tables 5–7 present the Bayes averages, ERs and BCIs of the unidentified parameters according to Type II censoring in cases 1 and 2, consequently.

Step 3: Repeat all of the preceding steps N times where N = 5000 is the number of repetitions.

Tables 5–7 show the simulation results of the Bayes estimates using BSEL and BLL functions for samples of size (n = 20, 60, 100). The values of the parameters for each sample size are chosen as (Case 1, ω=0.2,α=1.5,λ=0.9,θ=0.8,β=2.1,π_1=20%, π_2=20%,π_1=20%,π_2=40%$and$$\pi_1$=20%,$\pi_2$=70%) and (Case 2, $= 0.2, v = 3,\ \alpha = 0.5, \lambda = 0.6, \theta = 1.5, \beta = 3.5, \pi_1 = 20\%, \pi_2 = 20\%$).

5.2. Some applications

The major objective of this subsection is to show how the recommended methods can be applied in practice. This is achieved using two real-life data sets. [15] compared the efficiency of the OGNH-E to some well-known lifetime distributions, namely; *generalized linear exponential* (GLE), *exponentiated generalized linear exponential* (Ex-GLE), *exponential Lomax* (EL), *alpha power exponential* (APE) and *alpha power transformed Weibull* (APTW) distributions. They showed that the OGNH-E is fitted to the two real data sets using some measures of goodness of fit tests such as *Kolmogorov–Smirnov* (KS) statistic, *Anderson Darling* (AD) statistic, *Cramer-Von-Misses* (CM) statistic, *log-likelihood* (LL), *Akaike Information*

Table 6. Bayes averages, ERs and 95% Bayesian credible intervals for the parameters $\alpha, \lambda, \theta, \beta$ using informative prior under balanced linear exponential loss function based on Type II censoring (Case 1, N = 5000, $\omega = 0.2, v = 3, \alpha = 1.5, \lambda = 0.9, \theta = 0.8, \beta = 2.1 \pi_1 = 20\%, \pi_2 = 20\%, \pi_1 = 20\%, \pi_2 = 40\%$ and $\pi_1 = 20\%, \pi_2 = 70\%$).

n	π	Parameters	Averages	ERs	LL	UL	Length
		α	1.4966	0.0006	1.4904	1.5401	0.0497
20	$\pi_1 = 20\%$	λ	0.9794	0.0031	0.8775	0.9764	0.0989
	$\pi_2 = 20\%$	θ	0.8563	0.0037	0.7767	0.9094	0.1327
		β	1.9419	0.0010	2.0924	2.1549	0.0625
		α	1.3397	0.0399	1.1162	1.4605	0.3443
	$\pi_1 = 20\%$	λ	1.0279	0.0159	0.8612	1.0996	0.2384
	$\pi_2 = 40\%$	θ	0.7773	0.0062	0.6003	0.8317	0.2314
		β	1.7697	0.0592	1.6843	2.1618	0.4775
		α	1.6679	0.1138	1.2131	2.0702	0.8571
	$\pi_1 = 20\%$	λ	0.9257	0.0129	0.5974	1.0375	0.4401
	$\pi_2 = 70\%$	θ	0.8943	0.0244	0.5755	1.1132	0.5377
		β	1.8428	0.1358	1.1058	2.4009	1.2951
		α	1.4701	0.0002	1.4622	1.5003	0.0381
60	$\pi_1 = 20\%$	λ	0.9366	0.0002	0.8599	0.9119	0.0519
	$\pi_2 = 20\%$	θ	0.8221	0.0001	0.7814	0.8193	0.0379
		β	1.9030	0.0007	2.0533	2.1029	0.0496
		α	1.3805	0.0259	1.1451	1.5062	0.3610
	$\pi_1 = 20\%$	λ	0.8654	0.0141	0.6436	0.9455	0.3019
	$\pi_2 = 40\%$	θ	0.8443	0.0031	0.7265	0.9039	0.1775
		β	1.9777	0.0113	2.0349	2.2911	0.2561
		α	1.4058	0.0436	1.0337	1.6552	0.6215
	$\pi_1 = 20\%$	λ	0.9151	0.0065	0.7160	1.0269	0.3109
	$\pi_2 = 70\%$	θ	0.9018	0.0184	0.6677	1.0518	0.3841
		β	1.6066	0.2288	1.2823	2.1529	0.8706
		α	1.4789	2.6746E-6	1.4961	1.4999	0.0038
	$\pi_1 = 20\%$	λ	0.9402	6.0872E-7	0.8982	0.9014	0.0032
	$\pi_2 = 20\%$	θ	0.8194	2.0619E-6	0.7953	0.8007	0.0053
		β	1.9224	1.3047E-5	2.0995	2.1061	0.0065
		α	1.4793	0.0002	1.4757	1.5166	0.0409
100	$\pi_1 = 20\%$	λ	0.9556	0.0005	0.8995	0.9364	0.0369
	$\pi_2 = 40\%$	θ	0.8425	0.0009	0.7971	0.8567	0.0597
		β	1.9303	0.0004	2.0775	2.1356	0.0581
		α	1.4889	0.0002	1.4858	1.5293	0.0434
	$\pi_1 = 20\%$	λ	0.9662	0.0017	0.8881	0.9702	0.0821
	$\pi_2 = 70\%$	θ	0.7805	0.0031	0.7116	0.7916	0.0800
		β	1.9028	0.0009	2.0308	2.1152	0.0844

Table 7. Bayes averages, ERs and 95% Bayesian credible intervals for the parameters $\alpha, \lambda, \theta, \beta$ using informative prior under balanced square error loss and balanced linear exponential loss function based on Type II censoring (Case 2, $\mathrm{N} = 5000, \omega = 0.2, v = 3,\ \alpha = 0.5, \lambda = 0.6, \theta = 1.5, \beta = 3.5, \pi_1 = 20\%, \pi_2 = 20\%$).

n	π	Parameters	Averages	ERs	LL	UL	Length
		α	0.2534	0.3481	0.0000	0.4226	0.4226
20	$\pi_1 = 20\%$	λ	0.7730	0.0541	0.1096	1.0265	0.9169
	$\pi_2 = 20\%$	θ	1.6007	0.1007	1.4048	1.9907	0.5859
		β	2.8416	0.1010	2.4338	3.4979	1.0642
		α	0.5635	0.0859	0.0000	0.8231	0.8231
60	$\pi_1 = 20\%$	λ	0.8454	0.0507	0.4772	0.9567	0.4795
	$\pi_2 = 20\%$	θ	1.0442	0.1981	0.7657	1.3702	0.6045
		β	2.2314	1.4517	1.2815	3.3597	2.0782
		α	0.6419	0.0035	0.3613	0.4965	0.1351
100	$\pi_1 = 20\%$	λ	0.6413	0.0072	0.4609	0.5982	0.1372
	$\pi_2 = 20\%$	θ	1.3898	0.0007	1.4685	1.5576	0.0891
		β	2.9968	0.0035	3.3914	3.4963	0.1049
			LINEX loss function $v = 3$				
n	***π***	**Parameters**	**Averages**	**ERs**	**LL**	**UL**	**Length**
		α	0.5748	0.0322	0.0457	0.5436	0.4979
20	$\pi_1 = 20\%$	λ	0.7854	0.0323	0.4861	0.9418	0.4557
	$\pi_2 = 20\%$	θ	1.5306	0.0717	1.4099	2.0306	0.6207
		β	2.8634	0.0625	3.0431	3.4884	0.4453
		α	0.6606	8.0076E-4	0.4477	0.5034	0.0556
60	$\pi_1 = 20\%$	λ	0.7043	1.4647E-4	0.5791	0.6211	0.0421
	$\pi_2 = 20\%$	θ	1.3839	6.5720E-5	1.4878	1.5159	0.0281
		β	3.0248	5.0433E-4	3.4501	3.4989	0.0488
		α	0.6822	9.1006E-6	0.4996	0.5043	9.1006E-6
100	$\pi_1 = 20\%$	λ	0.6996	1.6221E-6	0.5966	0.6013	1.6221E-6
		θ	1.3808	1.5793E-6	1.4996	1.5022	1.5793E-6
	$\pi_2 = 20\%$	β	3.0371	1.5106E-5	3.4935	3.4994	1.5105E-5

Table 8. Bayes estimates for the parameters and their standard error for the real data sets based on Type II censoring.

Application I								
	n	Parameters	Estimates	SE		Parameters	estimates	SE
$\pi_1 = 20\%,\ \pi_2 = 20\%$		α	0.9423	0.0001	$\pi_1 = 20\%,\ \pi_2 = 70\%$	α	0.8982	0.2519
	50	λ	0.7235	0.0003		λ	0.8523	0.1208
		θ	0.6554	0.0008		θ	0.6542	0.1761
		β	1.1023	0.0006		β	1.2654	0.0246
			LINEX loss function $v = 3$					
	n	Parameters	Estimates	SE		Parameters	estimates	SE
$\pi_2 = 20\%\ \pi_1 = 20\%,$		α	0.9253	1.8221E-4	$\pi_1 = 20\%, \pi_2 = 70\%$	α	0.8999	0.0613
	50	λ	0.8235	1.1018E-4		λ	0.7592	0.0765
		θ	0.7562	9.0796E-5		θ	0.7623	0.0563
						β	1.2932	0.0246
		β	1.2325	6.1704E-4				
Application II								
	n	**Parameters**	**Estimates**	**SE**		**Parameters**	**estimates**	**SE**
$\pi_1 = 20\%, \pi_2 = 20\%$		α	1.2325	0.0002	$\pi_1 = 20\%,\ \pi_2 = 70\%$	α	1.1231	0.0368
	118	λ	0.9235	0.0003		λ	0.8932	0.0358
		θ	0.5624	0.0005		θ	0.6521	0.0189
		β	1.5213	0.0004		β	1.4325	0.0314
			LINEX loss function $v = 3$					
	n	**Parameters**	**Estimates**	**SE**		**Parameters**	**estimates**	**SE**
$\pi_1 = 20\%,\ \pi_2 = 20\%$		α	1.1325	7.4406E-5	$\pi_1 = 20\%,\ \pi_2 = 70\%$	α	1.2532	0.0111
	118	λ	0.8972	1.3708E-4		λ	0.9325	0.0056
		θ	0.6521	5.3342E-5		θ	0.7565	0.0054
		β	1.5232	4.0394E-4		β	1.5453	0.0314

Criterion (AIC), *Bayesian information criterion* (BIC), *corrected Akaike information criterion* (CAIC) and *Hannan-Quinn information criterion* (HQIC).

Application 1.

The first data presented by [21]. The data correspond to the time between failures for a repairable item. The data are: 0.036, 0.058, 0.061, 0.074, 0.078, 0.086, 0.102, 0.103, 0.114, 0.116, 0.148, 0.183, 0.192, 0.254, 0.262, 0.379, 0.381, 0.538, 0.570, 0.574, 0.590, 0.618, 0.645, 0.961, 1.228, 1.600, 2.006, 2.054, 2.804, 3.058, 3.076, 3.147, 3.625, 3.704, 3.931, 4.073, 4.393, 4.534, 4.893, 6.274, 6.816, 7.896, 7.904, 8.022, 9.337, 10.940, 11.020, 13.880, 14.730, and 15.080 (Table 1).

Application 2.

The second application represents the fracture toughness of Alumina (Al_2O_3) using data from the website: http://www.ceramics.nist.gov/srd/summary/ftmain.htm. The data are.

5.5, 5, 4.9, 6.4, 5.1, 5.2, 5.2, 5, 4.7, 4, 4.5, 4.2, 4.1, 4.56, 5.01, 4.7, 3.13, 3.12, 2.68, 2.77, 2.7, 2.36, 4.38, 5.73, 4.35, 6.81, 1.91, 2.66, 2.61, 1.68, 2.04, 2.08, 2.13, 3.8, 3.73, 3.71, 3.28, 3.9, 4, 3.8, 4.1, 3.9, 4.05, 4, 3.95, 4, 4.5, 4.5, 4.2, 4.55, 4.65, 4.1, 4.25, 4.3, 4.5, 4.7, 5.15, 4.3, 4.5, 4.9, 5, 5.35, 5.15, 5.25, 5.8, 5.85, 5.9, 5.75, 6.25, 6.05, 5.9, 3.6, 4.1, 4.5, 5.3, 4.85, 5.3, 5.45, 5.1, 5.3, 5.2, 5.3, 5.25, 4.75, 4.5, 4.2, 4, 4.15, 4.25, 4.3, 3.75, 3.95, 3.51, 4.13, 5.4,5, 2.1, 4.6, 3.2, 2.5, 4.1, 3.5, 3.2, 3.3, 4.6, 4.3, 4.3, 4.5, 5.5, 4.6, 4.9, 4.3, 3, 3.4, 3.7, 4.4, 4.9, 4.9, and 5 (Table 2).

The KS goodness of fit test is applied to check the validity of the fitted model. The P values are given, respectively, 0.5830 and 0.2101. The P value given in each case showed that the model fits the data.

The Bayes estimates and *standard error* (SE) of the unknown parameters for the real data sets based on Type II censoring are provided in Table 8 (Application 1 $n = 50$, $\pi_1 = 20\%, \pi_2 = 20\%$ *and* $\pi_1 = 20\%,\ \pi_2 = 70\%, \omega = 0.2,$) and (Application 2, $n = 118, \pi_1 = 20\%, \pi_2 = 20\%$ *and* $\pi_1 = 20\%$, $\pi_2 = 70\%$, $\omega = 0.2$). According to Table 8, the accuracy of the ERs for the parameters and acceleration factor generally improved as the percentage π of sample items assigned to accelerated conditions decreased. The Bayes estimates under the BLL functions have the smallest ERs when compared with their equivalent BSEL functions in Table 8.

5.3. Concluding remarks

(1) Tables 3−7 demonstrate that as sample size increases, the ML averages and Bayes balanced estimate are quite similar to the population parameter values. Furthermore when the sample size increasing the ERs are decreasing. This means that the estimates are consistent and get closer to the real parameter values as the sample size increases.
(2) As the sample size increases, the lengths of the CIs and BCIs of the parameters get shorter.
(3) Tables 5 and 6 demonstrated that the accuracy of the ERs for the parameters and acceleration factor generally improved as the percentage π of sample items assigned to accelerated conditions decreased.
(4) In most situations, the Bayes estimates under the BLL functions have the smallest ERs when compared with their equivalent BSEL functions.

6. General Conclusion

The measurement of product life using usual conditions frequently demands a lengthy period of time for products with a high level of reliability. Thus, ALT or PALT are employed to make it easier to estimate the units of reliability rapidly. Because ALT items are only processed under accelerated conditions, such relationships cannot be known or presumed in some cases. As a result, PALT is frequently employed in such cases; in PALT, items are performed under both usual and accelerated conditions. Based on Type II censoring, this study presented a SS-PALT. Considering that the lifetimes of test products have the OGNH-E distribution. The distribution parameter and the acceleration factor of the OGNH-E distribution are estimated using the ML and Bayesian methods. In Bayesian estimation the estimators are obtained using two different loss functions, the BSEL and BLL functions which are a symmetric and an asymmetric loss functions. The BLF is a mixture of Bayes and non-Bayes estimators. The performance of the proposed ML and Bayes estimates is evaluated through a simulation study and an application using real data sets. In general, numerical computations showed that as the acceleration factor increases the estimates of $(\alpha, \lambda, \theta)$ have been decreases. The ER, interval of the parameters and acceleration factor all decrease with sample size increases and the proportion sample (π) decreases. The Bayesian method for estimating the parameters of the OGNH-E distribution under SS-PALT using different types of loss functions such as general entropy and precautionary loss functions would be useful as a basis for future distribution theory research.

Conflicts of interest

Authors have declared that no competing interests exist.

References

[1] Chernoff H. Optimal accelerated life designs for estimation. Technimetrics 1962;4:381−408.

[2] Bessler S, Chernoff H, Marshall AW. An optimal sequential accelerated life test. Technimetrics 1962;4:367−79.

[3] Jian-Ping Z, Xin-Min G. Constant and step-stress accelerated life test of vacuum fluorescent display under Weibull distribution case. J Zhejiang Univ - Sci 2005;7:722−7.

[4] Wang B. Unbiased estimations for the exponential distribution based on step-stress accelerated life testing data. Appl Math Comput 2006;173:1227−37.

[5] Balakrishnan N, Han D. Exact inference for a simple step-stress model with competing risks for failure from exponential distribution under Type II censoring. J Stat Plann Inference 2008;138:4172−86.

[6] Wang BX. Interval estimation for exponential progressive Type-II censored step-stress accelerated life-testing. J Stat Plann Inference 2010;140:2706−18.

[7] Rezk HR, El-Helbawy AA, Al-Dayian GR. Statistical inference for a simple step-stress model based on censored data from Kumaraswamy Weibull distribution. Br J Math Comput Sci 2014;4:1496−514.

[8] Hakamipour N, Rezaei S. Optimizing the simple step stress accelerated life test with Frechet data. Revstat Stat. J 2017;15:1−23.

[9] Mohie El-Din MM, Abd El-Raheem AM, Abd El-Azeem SO. On step-stress accelerated life testing for power generalized Weibull distribution under progressive Type-II censoring. Ann. Data Sci 2021;8:629−44.

[10] Abdel-Hamid AH, Al-Hussaini EK. Step partially accelerated life tests under finite mixture models. J Stat Comput Simulat 2008;78:911−24.

[11] Ismail AA, Aly HM. Optimal planning of failure-step stress partially accelerated life tests under Type II censoring. J. Math. Anal 2009;3:1509−23.

[12] Ismail A. Planning step-stress life tests with Type II censored data. J. Sci. Res. Essay 2011;6:4021−8.

[13] El-Dessouky EA. Estimating costs of maintenance service policy using step stress partially accelerated life testing for the extension of exponential distribution under Type II censoring. Int. J. Contemp. Math. Sci 2017; 12:225−42.

[14] Aljohani HM, Alfar NM. Estimations with step-stress partially accelerated life tests for competing risks Burr XII lifetime model under Type-II censored data. Alex Eng J 2020; 59:1171−80.

[15] Ahmad Z, Elgarhy M, Hamedani GG, Butt NS. Odd generalized Nadarajah Haghighi generated family distributions with application to exponential model. Pak J Statistics Oper Res 2020;16:53−71.

[16] Zellner A. Bayesian and non-Bayesian estimation using balanced loss functions. In: Gupta SS, Burger JO, editors. Statistical decision theory and related topics. New York: Springer; 1994. 377−90.

[17] Ahmadi J, Jozani MJ, Marchand E, Parsian A. Bayes estimation based on k-record data from a general class of distributions under balanced type loss functions. J Stat Plann Inference 2009;139:1180−9.

[18] Al-Hussaini EK, Hussein M. Estimation under a finite mixture of exponentiated exponential components model and balanced square error loss. Open J Stat 2012;2:28−38.

[19] Abushal TA, Al-Zaydi AM. Inference on constant-partially accelerated life tests for mixture of Pareto distributions under progressive Type-II censoring. Open J Stat 2017;7:323−46.

[20] Al-Dayian GR, El-Helbawy AA, Refaey RM, Behairy SM. Bayesian estimation and prediction based on constant stress-partially accelerated life testing for Topp Leone-inverted Kumaraswamy distribution. J. Adv. Math 2021;36:11−32.

[21] Murthy DNP, Xie M, Jiang R. Weibull models. Hoboken, New Jersey: John Wiley and Sons, Inc; 2004.