[Al-Azhar Bulletin of Science](https://absb.researchcommons.org/journal)

Volume 34 | Issue 2 Article 8

2023

Section: Mathematics and Statistics

Soft Topological Notions Via Molodtsov Model

A.A. Nasef
Department of Physics and Engineering Mathematics, Faculty of Engineering , Kafrelsheikh University, Kafrelsheikh (33516), Egypt.

A.I. Aggour
Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City (11884), Cairo, Egypt.

A. Fathy
Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City (11884), Cairo, Egypt., ahmedfathy@azhar.edu.eg

S.M. Darwesh
Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City (11884), Cairo, Egypt.

How to Cite This Article

Nasef, A.A.; Aggour, A.I.; Fathy, A.; and Darwesh, S.M. (2023) "Soft Topological Notions Via Molodtsov Model," *Al-Azhar Bulletin of Science*: Vol. 34: Iss. 2, Article 8.
DOI: https://doi.org/10.58675/2636-3305.1648

ORIGINAL ARTICLE

Soft Topological Notions via Molodtsov Model

Arafa Abdel-zaher Nasef [a,*], Atef Ibrahim Aggour [b], Ahmed Fathy [b], Saad Mohamed Darwesh [b]

[a] Department of Physics and Engineering Mathematics, Faculty of Engineering, Kafrelsheikh University, Kafrelsheikh 33516, Egypt
[b] Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City 11884, Cairo, Egypt

Abstract

In the present paper, we introduce a new concept of soft sets called soft $g(\beta,\omega)$-closed sets. Also, we study the basic properties of this new concept and we investigate the relation between soft $g(\beta,\omega)$-closed sets and some of the other soft sets. Finally, we introduce the concept of soft $g(\beta,\omega)$-continuous map and we study the relationship between the new concept and some of the other types of soft continuity.

2010 Mathematics Subject Classification: 54B05, 54C08, 54D05

Keywords: Soft *g(β, ω)* closed set, Soft *g(β, ω)* -continuous map, Soft set

1. Introduction

Molodtsov [1] introduced the soft set in 1999. The soft sets were employed in application by Maji et al. Furthermore, soft information is a particular information class. Shabir and Naz explored some more fundamental features and introduced the soft topological space in [2]. Following that, some topological research discovered several fresh varieties of near soft open sets and investigated both their individual and interrelated characteristics. K. Kannan first discussed the idea of a soft generalised closed set in [3]. Many different soft generalised closed set types were then defined by various topologists. In this article, we introduced a brand-new class of soft generalised sets termed soft (β, ω)-closed and described its fundamental characteristics. Recent years have seen a significant growth in the number of articles regarding soft sets and their applications in numerous disciplines, as demonstrated in [4–6].

2. Preliminaries

In this section, we present the basic definition and some results of soft set theory. Let $\Re$ be a universal set and κ be the set of parameters, $P(\Re)$ is the power set of $\Re, A\subseteq\kappa$ and the soft set will be denoted by S-set.

Definition 2.1 [1]. A S-set (Ω, A) on $\Re$ is defined by the set of ordered pairs $(\Omega, A) = \{(\hbar, \Omega_A(\hbar)) : \hbar \in\kappa, \Omega_A(\hbar) \in P(V)\}$, where $\Omega_A : A \to P(\Re)$.

Definition 2.2 [1,7]. A S-set (Ω, A) is called null S-set if for all, $\hbar\in A$, then $\Omega(\hbar) = \varphi$ and its denoted by $\tilde{\phi}$. A S-set (Ω, A) is called absolute S-set if for all $a\in A, \Omega(\hbar) = \Re$ and its denoted by $\tilde{\Re}$.

Definition 2.3 [1,7]. Let (Ω, A) and (Ψ, β) be two S-sets over $\Re$. Then, the union of (Ω, A) and (Ψ, β) is a S-set (H, C) where $C = (A\cup\beta)$ and $H(\hbar) = \Omega(\hbar)$ if $\hbar\in A-\beta, H(\hbar) = \Psi(\hbar)$ if $\hbar\in\beta - A, H(\hbar) = \Omega(\hbar)\cup\Psi(\hbar)$ if $\hbar\in A\cap\beta$.

Definition 2.4. [1,7] Let (Ω, A) and (Ψ, β) be two S-sets over $\Re$. The intersection of (Ω, A) and (Ψ, β) is a S-set (F, D) where $D = A\cap\beta, F(\hbar) = \Omega(\hbar)\cap\Psi(\hbar)$, for all $\hbar\in D$.

Definition 2.5. [1,7] A S-set (Ω, A) is called a S-subset of (Ψ, β) if $A\subseteq\beta$ and $\Omega(\hbar)\subseteq\Psi(\hbar)$ for all $a\in A$. We write $(\Omega, A)\tilde{\subseteq}(\Psi, \beta)$.

Definition 2.6 [1,7]. Let $(\Omega, \kappa), (\Psi, \kappa)$ be two S-sets over $\Re$. Then, the difference of $(\Omega, \kappa), (\Psi, \kappa)$ is denoted by $(H, C) = (\Omega, \kappa)\setminus(\Psi, \kappa)$ such that $H(C) = \Omega(\hbar)\setminus\Psi(\hbar)$ for all $\hbar$ in A.

Definition 2.7. [1,7] The relative complement of (Ω, A) is denoted by $(\Omega, A)^c = (\Omega^c, A)$ where $\Omega^c : A \to P(\Re)$ given by $\Omega^c(\hbar) = \Re\setminus\Omega(\hbar)$ for all $\hbar$ in A.

Received 18 June 2023; revised 25 July 2023; accepted 27 July 2023.
Available online 13 October 2023

* Corresponding author.
E-mail address: ahmedfathy@azhar.edu.eg (A.A.-z. Nasef).

https://doi.org/10.21608/2636-3305.1648

Definition 2.8 [2,8]. Let τ be a collection of S-sets over $\Re$. Then, τ is called a S-topology on $\Re$ if the following axioms are satisfied:

(1) $\tilde{\varphi}, \tilde{\Re} \in \tau$.
(2) The union of arbitrary S-sets in τ belongs to τ
(3) The intersection of two S-sets in τ belongs to τ.

The triple $(\Re, \tau, \kappa)$ is called a S-topological space and the members of τ are called S-open sets and its complement are called S-closed sets.

Definition 2.9. [2,8] The S-interior of (Ω, κ) is the union of all S-open sets of topological space $(\Re, \tau, \kappa)$ contained in (Ω, κ) and its denoted by $\mathrm{int}(\Omega, \kappa)$.

Definition 2.10. [2,8] The S-closure of (Ω, κ) is the intersection of all S-closed sets containing and its denoted by $cl(\Omega, \kappa)$.

Definition 2.11. [6,9–11] Let $(\Re, \tau, \kappa)$ be a S-topological space. Then, (Ω, κ) is said to be:

(1) A S-α-open set if $(\Omega, \kappa)\tilde{\subseteq}\mathrm{int}(cl(\mathrm{int}(\Omega, \kappa)))$.
(2) A S-semi-open set if $(\Omega, \kappa)\tilde{\subseteq}cl(\mathrm{int}(\Omega, \kappa))$
(3) A S-pre open set if $(\Omega, \kappa)\tilde{\subseteq}\mathrm{int}(cl(\Omega, \kappa))$.
(4) A S-b-open set if $(\Omega, \kappa)\tilde{\subseteq}\mathrm{int}(cl(\Omega, \kappa))\cup cl(\mathrm{int}(\Omega, \kappa))$.
(5) A S-β-open set if $(\Omega, \kappa)\tilde{\subseteq}cl(\mathrm{int}(cl(\Omega, \kappa)))$.

The family of all S-α-open (resp. S-semi-open, S-pre open, S-b-open and S-β-open) sets in a S-topological space $(\Re, \tau, \kappa)$, is denoted by $S\alpha O$ (resp. SSO, SPO, SbO and $S\beta O$.

Definition 2.12. [6,9–11] A S-set (Ω, κ) of a S-topological space $(\Re, \tau, \kappa)$ is called S-α-closed (resp. S-semi-closed, S-pre closed, S-b-closed and S-β-closed) sets if its complements is S-α-open (resp. S-semi-open, S-b-open and S-β-open) sets.

Definition 2.13. [6,9–11] Let $(\Re, \tau, \kappa)$ be a S-topological space and (Ω, κ) be a S-set. Then, The, intersection of all S-α-closed (resp. S-semi-closed, S-pre closed, S-b-closed and S-β-closed) sets containing (Ω, κ) is called S-α-closure (resp. S-semi-closure, S-pre closure, S-b-closure and S-β-closure) of (Ω, κ) and its denoted by $S\alpha cl(\Omega, \kappa)$ (resp. $SScl(\Omega, \kappa)$, $SPcl(\Omega, \kappa)$,

(1) $Sbcl(\Omega, \kappa)$ and $S\beta cl(\Omega, \kappa)$.
(2) The union of all S-α-open (resp. S-semi-open, S-pre open, S-b-open and S-β-open) sets containing in (Ω, κ) is called S-α-interior (resp. S-semi-interior, S-pre interior, S-b- interior and S-β-interior) of (Ω, κ) and it is denoted by $S\alpha\mathrm{int}(\Omega, \kappa)$ (resp. $SS\mathrm{int}(\Omega, \kappa)$, $Sp\mathrm{int}(\Omega, \kappa)$, $Sb\mathrm{int}(\Omega, \kappa)$ and $S\beta\mathrm{int}(\Omega, \kappa)$.

Definition 2.14. [12–14] A S-subset (Ω, κ) of a S-topological space $(\Re, \tau, \kappa)$ is called:

(1) A S-generalized closed set (sg-closed) if $(\Omega, \kappa)\subseteq(\Psi, \kappa)$ and (Ψ, κ) is S-open implies that $cl(\Omega, \kappa)\subseteq(\Psi, \kappa)$
(2) A S-semi-generalized closed set (SSg-closed set) if $(\Omega, \kappa)\subseteq(\Psi, \kappa)$ and (Ψ, κ) is S-semi-open implies that $SScl(\Omega, \kappa)\subseteq(\Psi, \kappa)$
(3) A generalized S-semi-closed set (SgS-closed set) if $(\Omega, \kappa)\subseteq(\Psi, \kappa)$ and (Ψ, κ) is S-open implies that $SScl(\Omega, \kappa)\subseteq(\Psi, \kappa)$
(4) A S-α-generalized closed set ($S\ \alpha\ g$-closed set) if $(\Omega, \kappa)\subseteq(\Psi, \kappa)$ and (Ψ, κ) is S-α-open implies that $SScl(\Omega, \kappa)\subseteq(\Psi, \kappa)$
(5) A S-generalized α-closed set ($Sg\ \alpha$-closed set) if $(\Omega, \kappa)\subseteq(\Psi, \kappa)$ and (Ψ, κ) is S-open implies that $S\alpha cl(\Omega, \kappa)\subseteq(\Psi, \kappa)$.
(6) A S-α-closed set ($S\ \omega$-closed set) if $(\Omega, \kappa)\subseteq(\Psi, \kappa)$ and (Ψ, κ) is S-semi-open set implies that $Scl(\Omega, \kappa)\subseteq(\Psi, \kappa)$.
(7) A S-generalized pre closed set (Sgp-closed set) if $(\Omega, \kappa)\subseteq(\Psi, \kappa)$ and (Ψ, κ) is S-open set implies that $Spcl(\Omega, \kappa)\subseteq(\Psi, \kappa)$.

Definition 2.15. [15] A map $\mho$: $(\Re, \tau, \kappa)$ (V, τ', κ') is called:

(i) A S-continuous map if $\mho^{-1}(\Omega', \kappa')$ is a S-open set in $(\Re, \tau, \kappa)$, for every S-open set (Ω', κ') in (V, τ', κ').
(ii) A S-α- continuous map if $\mho^{-1}(\Omega', \kappa')$ is a α-S-open set in $(\Re, \tau, \kappa)$, for every S-open set (Ω', κ') in (V, τ', κ').
(iii) A S-semi continuous map if $\mho^{-1}(\Omega', \kappa')$ is a S-semi-open set in $(\Re, \tau, \kappa)$, for every S-open set (Ω', κ') in (V, τ', κ').

3. A Soft generalized (β, ω)-closed set

Definition 3.1. Let $(\Re, \tau, \kappa)$ be a S-topological space. If $(\Omega, \kappa)\subseteq(\Re, \kappa)$ and $(\Re, \kappa)$ is S-ω-open set implies that $\beta cl(\Omega, \kappa)\subseteq\mathrm{int}(\Re, \kappa)$, then (Ω, κ) is called a S-generalized $g(\beta, \omega)$ closed set. The set of all S-generalized $g(\beta, \omega)$ closed sets is denoted by $Sg(\beta, \omega)c$.

In this paper, we consider $\Re = \{\Re_1, \Re_2\}$ and $= \{\hbar_1, \hbar_2\}, (\Omega, \kappa) = \tilde{\Re} = \{(\hbar_1, \Re), (\hbar_2, \Re)\}, (\Omega_2, \kappa) = \tilde{\phi} = \{(\hbar_1, \phi), (\hbar_2, \phi)\}, (\Omega_3, \kappa) = \{(\hbar_1, \{\Re_1\}), (\hbar_2, \{\Re_1\})\}$, $(\Omega_4, \kappa) = \{(\hbar_1, \{\Re_2\}), (\hbar_2, \{\Re_2\})\}$, $(\Omega_5, \kappa) = \{(\hbar_1, \Re), (\hbar_2, \phi)\}$,

$(\Omega_6, \kappa) = \{(\hbar_1, \Re), (\hbar_2, \Re_1)\}$, $(\Omega_7, \kappa) = \{(\hbar_1, \Re), (\hbar_2, \{\Re_2\})\}$, $(\Omega_8, \kappa) = \{(\hbar_1, \phi)$,

$(\hbar_2, \{\Re_1\})\}$, $(\Omega_9, \kappa) = \{(\hbar_1, \phi), (\hbar_2, \{\Re_2\})\}$, $(\Omega_{10}, \kappa) = \{(\hbar_1, \{\Re_1\}), (\hbar_2, \{\Re_2\})\}$,

$(\Omega_{11}, \kappa) =$
$\{(\hbar_1, \phi), (\hbar_2, \Re)\}$, $(\Omega_{12}, \kappa) = \{(\hbar_1, \{\Re_1\}), (\hbar_2, \Re)\}$,
$(\Omega_{13}, \kappa) = \{(\hbar_1, \{\Re_2\}),$

$(\hbar_2, \Re)\}, (\Omega_{14}, \kappa) = \{(\hbar_1, \{\Re_1\}), (\hbar_2, \phi)\}$, $(\Omega_{15}, \kappa) = \{(\hbar_1, \{\Re_2\}), (\hbar_2, \phi)\}$,

$(\Omega_{16}, \kappa) = \{(\hbar_1, \{\Re_2\}), (\hbar_2, \{\Re_2\})\}$

Example 3.1. Let $\tau = \{\tilde{\Re}, \tilde{\phi}, \Omega_3, \Omega_6, \Omega_{13}\}$. Then, $\tau^c = \{\tilde{\Re}, \tilde{\phi}, \Omega_4, \Omega_9, \Omega_{15}\}$. We have that $g(\beta, \omega)c = \{\Omega_1, \Omega_2, \Omega_4, \Omega_5, \Omega_7, \Omega_9, \Omega_{11}, \Omega_{13}, \Omega_{15}, \Omega_{16}\}$.

Proposition 3.1. A S-set (Ω, κ) is S-ω-open if and inly if $(\Psi, \kappa)\tilde{\subseteq}\mathrm{int}(\Omega, \kappa)$, whenever (Ψ, κ) is S-semi-closed and $(\Psi, \kappa)\tilde{\subseteq}(\Omega, \kappa)$.

Proof. Let (Ω, κ) be S-ω-open. Then (Ω^c, κ) is a S-ω-closed set. So, $cl(\Omega^c, \kappa)\tilde{\subseteq}(\Psi, \kappa)$, whenever $(\Omega^c, \kappa)\tilde{\subseteq}(\Psi_1, \kappa)$, where (Ψ_1, κ) is S-semi-open. Hence, (Ψ_1^c, κ) is S-semi-closed and so $\mathrm{int}(cl(\Psi^c, \kappa))\tilde{\subseteq}(\Psi^c, \kappa)$. If we assume that $\Psi_1^c = \Psi$, then $\mathrm{int}(cl(\Psi, \kappa))\tilde{\subseteq}(\Psi, \kappa)$. Since $cl(\psi^c, \kappa)\tilde{\subseteq}(\psi, \kappa)$, then $(\psi, \kappa)\tilde{\subseteq}\mathrm{int}(\Omega, \kappa)$. Conversely, let (Ω, κ) be a S-set such that $(\psi, \kappa)\tilde{\subseteq}\mathrm{int}(\Omega, \kappa)$ whenever (ψ, κ) is S-semi-closed and $(\psi, \kappa)\tilde{\subseteq}(\Omega, \kappa)$. Then, $(\Omega^c, \kappa)\tilde{\subseteq}(\psi^c, \kappa)$, wherever (ψ^c, κ) is S-semi-open set implied that $(\Omega^c, \kappa)\tilde{\subseteq}(\psi^c, \kappa)\tilde{\subseteq}cl(\mathrm{int}(\psi^c, \kappa))$ then $cl(\Omega^c, \kappa)\subseteq cl(\mathrm{int}(\psi^c, \kappa))$. Since

$[cl(\mathrm{int}(\psi^c, \kappa))]$ is semi-open set, then (Ω^c, κ) is S-ω-closed set and so (Ω, κ) is S-ω-open.

Proposition 3.2. Every S-semi-closed set is a $Sg(\beta, \omega) - C$ set.

Proof. Let (ψ, κ) be a S-semi-closed set and $(\psi, \kappa)\tilde{\subseteq}(\Omega, \kappa)$, where (Ω, κ) is S-ω-open set. By Proposition 3.1, we have that $(\psi, \kappa)\tilde{\subseteq}\mathrm{int}(\Omega, \kappa)$. Also, since (ψ, κ) is S-semi-closed, then (ψ, κ) is S-β-closed, and so

$S\beta cl(\psi, \kappa) = (\psi, \kappa)\tilde{\subseteq}\mathrm{int}(\Omega, \kappa)$. Hence (ψ, κ) is a $Sg(\beta, \omega) - C$ set.

We not that the converse of the proposition 3.2 may not be a true, in general as shown in Example 3.1.

Example 3.2. Continue to Example 3.1, we have that S-sets.

$\{\Omega_5, \Omega_7, \Omega_{10}, \Omega_{11}, \Omega_{13}, \Omega_{14}, \Omega_{16}\}$ are $S-(\beta, \omega)$-closed sets but not S-semi-closed sets.

Proposition 3.3. Every S-closed set is a $Sg(\beta, \omega) - C$ set.

Proof. Let (ψ, κ) be a S-closed set and $(\psi, \kappa)\subseteq(\Omega, \kappa)$, where (Ω, κ) is S-ω-open. Then, by Proposition 3.1 $(\psi, \kappa)\subseteq\mathrm{int}(\Omega, \kappa)$. Since (ψ, κ) is a S-closed set, then (ψ, κ) is a S-β-closed set and then $S\beta cl(\psi, \kappa) = (\psi, \kappa)\subseteq \mathrm{int}(\Omega, \kappa)$. Hence, (ψ, κ) is $Sg(\beta, \omega) - C$

We note that the converse of the proposition 3.3 may not be a true, in general as shown in Example 3.1.

Example 3.3. Continue to Example 3.1, we have that the S-sets.

$\{\Omega_5, \Omega_7, \Omega_{10}, \Omega_{11}, \Omega_{13}, \Omega_{16}\}$ are $Sg(\beta, \omega) - C$ sets but not S-closed sets.

Proposition 3.4. Every S-α-closed set is a $Sg(\beta, \omega) - C$ set.

Proof. Let (ψ, κ) be a S-α-closed set and $(\psi, \kappa)\subseteq(\Omega, \kappa)$, where (Ω, κ) is S−W-open. Then, by Lemma 3.1 we have that $(\psi, \kappa)\subseteq\mathrm{int}(\Omega, \kappa)$. Since (ψ, κ) be S-α-closed, then (ψ, κ) is a S-β-closed and $S\beta cl(\psi, \kappa) = (\psi, \kappa)\subseteq\mathrm{int}(\Omega, \kappa)$. Hence, (ψ, κ) is $Sg(\beta, \omega) - C$ set.

We note that the converse of the proposition 3.4 may not be a true, in general as shown in Example 3.1.

Example 3.4. Continue to Example 3.1, we have that the S-set Ω_5 is $Sg(\beta, \omega) - C$ set but not S-α-closed.

Proposition 3.5. Arbitrary intersection of $Sg(\beta, \omega) - C$ sets is also $Sg(\beta, \omega) - C$ set.

Proof. Let (Ω_λ, κ) be $Sg(\beta, \omega) - C$ sets in the S-topological space $(\Re, \tau, \kappa)$. Then, $S\beta cl(\Omega_\lambda, \kappa)\subseteq\mathrm{int}(\Re_\lambda, \kappa)$, for each λ, whenever $(\Omega_\lambda, \kappa)\subseteq(\Re_\lambda, \kappa)$, are S-$\omega$-open sets. Hence, we have that $\cap_\lambda S\beta cl(\Omega_\lambda, \kappa)\subseteq\cap_\lambda\mathrm{int}(\Re_\lambda, \kappa)$, for each λ, whenever $\cap_\lambda(\Omega_\lambda, \kappa)\subseteq(\Re_\lambda, \kappa)$. Since $\cap_\lambda(\Omega_\lambda, \kappa)$ is S-ω-open, then $\cap_\lambda(\Omega_\lambda, \kappa)$ is a $Sg(\beta, \omega) - C$ set.

Remark 3.1. The S-union of two $Sg(\beta, \omega) - C$ sets may not be a $Sg(\beta, \omega) - C$ set. This is clear from the following example.

Example 3.5. Continue to Example 3.1, we have that Ω_5, Ω_{16} are $S(\beta, \omega) - C$ sets but $\Omega_5\cup\Omega_{16} = \Omega_6 \notin Sg(\beta, \omega) - C$.

Remark 3.2. The concept of S-generalized (β, ω) closed set and S-β-closed sets are independent. This is clear from the following example.

Example 3.6. Continue to Example 3.1, we have that Ω_8 is S-β- closed set but not generalized (β, ω)-closed set.

Example 3.7. Continue to Example 3.1, let $\tau = \{\Omega_1, \Omega_2, \Omega_5\}$. Then, we have that $\tau^c = \{\Omega_1, \Omega_2, \Omega_4\}\{\Omega_5, \Omega_6, \Omega_7\}$ are S-generalized (β, ω)-closed set but int S-β-closed set.

Remark 3.3. The concept of S-generalized β-closed set and S-closed sets are independent. This is clear from the following Example.

Example 3.8. Continue to Example 3.1, we have that Ω_8 is a S-pre closed set but its not S-generalized (β, ω)-closed set.

Example 3.9. Continue to Example 3.1, we have get $\Omega_5, \Omega_6, \Omega_7$ are S-generalized (β, ω)-closed set nut not S-pre closed set.

Remark 3.4. The concept of S-g-closed sets and generalized (β, ω)-closed sets are independent. This is clear from the following Example.

Example 3.10. Continue to Example 3.1, we have that $\Omega_4, \Omega_5, \Omega_7, \Omega_{10}, \Omega_{11}, \Omega_{13}$, $\Omega_{14}, \Omega_{15}, \Omega_{16}$ are generalized (β, ω)-closed sets but not generalized closed sets.

The same example above, say that Ω_{12}, Ω_{16} are S-generalized closed sets but not S-generalized (β, ω)-closed set.

Remark 3.5. The concept of S-generalized (β, ω)-closed set and S-ω-closed set are independent this is clear form the following Example.

Example 3.11. Continue to Example 3.1, we have that $\Omega_5, \Omega_{10}, \Omega_{11}, \Omega_{14}, \Omega_{16}$ are S-generalized (β, ω)-closed set but not S-ω-closed set.

Example 3.12. Continue to Example 3.1, let $\tau = \{\Omega_1, \Omega_2\}, \tau^c = \{\Omega_1, \Omega_2\}$ we have that, $\{\Omega_3, \Omega_4, \Omega_{16}\}$ are S-ω-closed set but not S-generalized (β, ω)-closed set.

Remark 3.6. The concept of S-generalized (β, ω)-closed set and S-α-generalized closed set are independent is clear form the following Example.

Example 3.13. Continue to Example 3.1, we have that $\Omega_5, \Omega_7, \Omega_{10}, \Omega_{11}, \Omega_{13}, \Omega_{14}, \Omega_{16}$ are S-generalized (β, ω)-closed set but not S-α-generalized closed set.

Example 3.14. Continue to Example 3.1, we have that $(\Omega_3, ..., \Omega_{16})$ are S-α-generalized closed sets but S-generalized (β, ω)-closed sets.

Remark 3.7. The concept of S-generalized (β, ω)-closed set and S-generalized semi-closed set are independent.

Example 3.15. Continue to Example 3.1, we have that $\Omega_{10}, \Omega_{11}, \Omega_{14}, \Omega_{16}$ are generalized (β, ω)-closed set but not S-generalized semi-closed set.

Example 3.16. Continue to Example 3.1, let $\{\tau = \{\Omega_1, \Omega_2, \Omega_{10}, \Omega_{16}\}$. We have that $\{\Omega_3, \Omega_4, \Omega_5, \Omega_6, \Omega_7, \Omega_8, \Omega_9, \Omega_{10}, \Omega_{11}, \Omega_{12}, \Omega_{13}, \Omega_{14}\}$ are generlaized S-semi-closed but not S-generalized (β, ω)-closed

4. Soft generalized (β, ω)-continuous map

Definition 4.1. A map $\mho$: $(\Re, \tau, \kappa)$ (V, τ', κ') is called a S-generalized (β, ω)-continuous map if $\mho^{-1}(\Omega', \kappa')$ is S-generalized (β, ω)-open set in $(\Re, \tau, \kappa)$, for every S-open set (Ω', κ') in (V, τ', κ').

Example 4.1. Let $\mho$: $(\Re, \tau, \kappa)$ (V, τ', κ') be a map, where $(\Re, \tau, \kappa)$ is a S-topological space defined in Example 3.1 and Let $V = \{a, b\}, \kappa' = \{\hbar_1', \hbar_2'\}$ and $(V, \tau', \kappa') = \{V, \phi, \Omega_1', \Omega_2', \Omega_3'\}$ such that $\Omega_1' = \{(\hbar_1', \{a\}), (\hbar_2', \{a\})\}$, $\Omega_2' = \{(\hbar_1', V), (\hbar_2', \{a\})\}$ and $\Omega_3' = \{(\hbar_1', \{a\}), (\hbar_2', V)$. Then $\mho$ is a S-generalized (β, ω)-continuous map.

Theorem 4.1. The identity map $I : (\Re, \tau, \kappa) \to (\Re, \tau, \kappa)$ is a S-generalized (β, ω)-continuous map.

Proof. It is obvious.

Theorem 4.2. Let $\mho$: $(\Re, \tau, \kappa)$ (V, τ', κ') be a S-map. Then,

(1) If $\mho$ is S-continuous, then $\mho$ is S-generalized (β, ω)-continuous
(2) If $\mho$ is S-α-continuous, then $\mho$ is S-generalized (β, ω)-continuous
(3) If $\mho$ is S-semi-continuous, then $\mho$ is S-generalized (β, ω)-continuous

Proof. It is obvious.

Conflicts of interest

The authors approve that no conflict of interest.

References

[1] Molodtsov D. Soft Theory first results. Comput Math Appl 1999;37:19−31.
[2] Shabir M, Naz M. On soft topological spaces. Comput Math Appl 2011;61:1786−99.
[3] Kannan K. Soft generalized closed sets in soft topological spaces. J Theor Appl Inf Technol 2012;37:17−20.
[4] Nasef AA, Parimale M, Jeevith R, EL-Sayed MK. Soft ideal theory and applications. Int J Nonlinear Anal Appl 2022;13: 1−10.
[5] Nasef AA, Azam AA. α− Completely regular and almost α− Completely regular spaces. Math Probl Eng 2022;467466:1−6.
[6] Alzagrani S, Nasef AA, Youns N, EL-Maghrabi AI, Badr MS. Soft toplogical approaches via soft γ−open sets. AIHS Math 2022;7:12144−53.
[7] Maji PK, Biwas R, Roy R. Soft set theory. Comput Math Appl 2003;45:555−62.
[8] Cagman N, Karats S, Enginoglu S. Soft topology. Comput Math Appl 2011;62:351−8.
[9] Chen B. Soft semi-open sets and related properties in soft topological spaces. Appl Inf Sci 2013;7:287−94.
[10] Arokia Rani I, Talbinaa T. A soft generalized pre closed sets and space. Proc IGMSCA 2014:138−87.
[11] Arokia Rani I, Lancy AA. Soft gβ closed sets and soft gsβ closed sets in soft topological spaces. Int J Math Arch 2013;4: 17−23.
[12] Nadhini T, Kalaiselvi A. Soft ˆg closed sets in topological spaces. Int J Innov Sci Eng Technol 2014;3:14595−600.
[13] Guzel ZE, Yskel S, Tozlu N. On soft generalized preregular closed and open sets in soft topological spaces. Appl Math Sci 2014;8:7875−84.
[14] Veerakumar MKRS. On Soft ˆg closed sets and ˆg LC functions. Indian J Math 2001;43:231−47.
[15] Kharaland A, Ahmed B. Mapping on soft classes. New Math Comput 2017;3:471−81.