Soft Topological Notions Via Molodtsov Model

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How to Cite This Article
DOI: https://doi.org/10.58675/2636-3305.1648

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Soft Topological Notions via Molodtsov Model

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Abstract

In the present paper, we introduce a new concept of soft sets called soft \( g(\beta, \omega) \)-closed sets. Also, we study the basic properties of this new concept and we investigate the relation between soft \( g(\beta, \omega) \)-closed sets and some of the other soft sets. Finally, we introduce the concept of soft \( g(\beta, \omega) \)-continuous map and we study the relationship between the new concept and some of the other types of soft continuity.

Keywords: Soft \( g(\beta, \omega) \) closed set, Soft \( g(\beta, \omega) \) -continuous map, Soft set

1. Introduction

Molodtsov [1] introduced the soft set in 1999. The soft sets were employed in application by Maji et al. Furthermore, soft information is a particular information class. Shabir and Naz explored some more fundamental features and introduced the soft topological space in [2]. Following that, some topological research discovered several fresh varieties of near soft open sets and investigated both their individual and interrelated characteristics. K. Kannan first discussed the idea of a soft generalised closed set in [3]. Many different soft generalised closed set types were then defined by various topologists. In this article, we introduced a brand-new class of soft generalised sets termed soft \( (\beta, \omega) \)-closed and described its fundamental characteristics. Recent years have seen a significant growth in the number of articles regarding soft sets and their applications in numerous disciplines, as demonstrated in [4–6].

2. Preliminaries

In this section, we present the basic definition and some results of soft set theory. Let \( \mathcal{R} \) be a universal set and \( \kappa \) be the set of parameters, \( P(\mathcal{R}) \) is the power set of \( \mathcal{R} \), \( A \subseteq \kappa \) and the soft set will be denoted by S-set.

Definition 2.1 [1]. A S-set \( (\Omega, A) \) on \( \mathcal{R} \) is defined by the set of ordered pairs \( (\Omega, A) = \{(h, \Omega_A(h)) : h \in \kappa, \Omega_A(h) \in P(V)\} \), where \( \Omega_A : A \rightarrow P(\mathcal{R}) \).

Definition 2.2 [1,7]. A S-set \( (\Omega, A) \) is called null S-set if for all, \( h \in A \), then \( \Omega(h) = \emptyset \) and its denoted by \( \phi \). A S-set \( (\Omega, A) \) is called absolute S-set if for all \( a \in A \), \( \Omega(h) = \mathcal{R} \) and its denoted by \( \mathcal{R} \).

Definition 2.3 [1,7]. Let \( (\Omega, A) \) and \( (\Psi, \beta) \) be two S-sets over \( \mathcal{R} \). Then, the union of \( (\Omega, A) \) and \( (\Psi, \beta) \) is a S-set \( (H, C) \) where \( C = (A \cup \beta) \) and \( H(h) = \Omega(h) \) if \( h \in A \cup \beta \), \( H(h) = \Psi(h) \) if \( h \in \beta - A \cup \beta \).

Definition 2.4. [1,7] Let \( (\Omega, A) \) and \( (\Psi, \beta) \) be two S-sets over \( \mathcal{R} \). The intersection of \( (\Omega, A) \) and \( (\Psi, \beta) \) is a S-set \( (F, D) \) where \( D = A \cap \beta \), \( F(h) = \Omega(h) \cap \Psi(h) \), for all \( h \in D \).

Definition 2.5. [1,7] A S-set \( (\Omega, A) \) is called a S-subset of \( (\Psi, \beta) \) if \( A \subseteq \beta \) and \( \Omega(h) \subseteq \Psi(h) \) for all \( h \in A \). We write \( (\Omega, A) \subseteq (\Psi, \beta) \).

Definition 2.6. [1,7] Let \( (\Omega, \kappa), (\Psi, \kappa) \) be two S-sets over \( \mathcal{R} \). Then, the difference of \( (\Omega, \kappa), (\Psi, \kappa) \) is denoted by \( (H, C) = (\Omega, \kappa) \setminus (\Psi, \kappa) \) such that \( H(h) = \Omega(h) \setminus \Psi(h) \) for all \( h \in \kappa \).

Definition 2.7. [1,7] The relative complement of \( (\Omega, A) \) is denoted by \( (\Omega, A)^c = (\Omega^c, A) \) where \( \Omega^c : A \rightarrow P(\mathcal{R}) \) given by \( \Omega^c(h) = \mathcal{R} \setminus \Omega(h) \) for all \( h \in A \).
Definition 2.8 [2,8]. Let \( \tau \) be a collection of S-sets over \( \mathcal{H} \). Then, \( \tau \) is called a S-topology on \( \mathcal{H} \) if the following axioms are satisfied:

1. \( \emptyset, \mathcal{H} \in \tau \).
2. The union of arbitrary S-sets in \( \tau \) belongs to \( \tau \).
3. The intersection of two S-sets in \( \tau \) belongs to \( \tau \).

The triple \( (\mathcal{H}, \tau, k) \) is called a S-topological space and the members of \( \tau \) are called S-open sets and its complement are called S-closed sets.

Definition 2.9. [2,8] The S-interior of \( (\Omega, k) \) is the union of all S-open sets of topological space \( (\mathcal{H}, \tau, k) \) contained in \( (\Omega, k) \) and its denoted by \( \text{int}(\Omega, k) \).

Definition 2.10. [2,8] The S-closure of \( (\Omega, k) \) is the intersection of all S-closed sets containing and its denoted by \( cl(\Omega, k) \).

Definition 2.11. [6,9–11] Let \( (\mathcal{H}, \tau, k) \) be a S-topological space. Then, \( (\Omega, k) \) is said to be:

1. A S-\( \alpha \)-open set if \( (\Omega, k) \subseteq \text{int}(cl(\text{int}(\Omega, k))) \).
2. A S-semi-open set if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\Omega, k)) \).
3. A S-pre open set if \( (\Omega, k) \subseteq \text{int}(\text{cl}(\Omega, k)) \).
4. A S-\( b \)-open set if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\text{cl}(\Omega, k))) \).
5. A S-\( \beta \)-open set if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\text{cl}(\Omega, k))) \).

The family of all S-\( \alpha \)-open (resp. S-semi-open, S-pre open, S-\( b \)-open and S-\( \beta \)-open) sets in a S-topological space \( (\mathcal{H}, \tau, k) \), is denoted by SaO (resp. SSO, SPO, SBO and SBO).

Definition 2.12. [6,9–11] A S-set \( (\Omega, k) \) of a S-topological space \( (\mathcal{H}, \tau, k) \) is called S-\( \alpha \)-closed (resp. S-semi-closed, S-pre closed, S-\( b \)-closed and S-\( \beta \)-closed) sets if its complements is S-\( \alpha \)-open (resp. S-semi-open, S-pre-open and S-\( b \)-open) sets.

Definition 2.13. [6,9–11] Let \( (\mathcal{H}, \tau, k) \) be a S-topological space and \( (\Omega, k) \) be a S-set. Then, the intersection of all S-\( \alpha \)-closed (resp. S-semi-closed, S-pre closed, S-\( b \)-closed and S-\( \beta \)-closed) sets containing \( (\Omega, k) \) is called S-\( \alpha \)-closure (resp. S-semi-closure, S-pre closure, S-\( b \)-closure and S-\( \beta \)-closure) of \( (\Omega, k) \) and its denoted by \( \text{Sacl}(\Omega, k) \) (resp. SScl(\( \Omega, k \)), SNMP(\( \Omega, k \)), SShint(\( \Omega, k \)) and SShint(\( \Omega, k \)).

(1) S\text{Sacl}(\( \Omega, k \)) and S\text{Shint}(\( \Omega, k \)).

(2) The union of all S-\( \alpha \)-open (resp. S-semi-open, S-pre open, S-\( b \)-open and S-\( \beta \)-open) sets containing in \( (\Omega, k) \) is called S-\( \alpha \)-interior (resp. S-semi-interior, S-pre interior, S-\( b \)-interior and S-\( \beta \)-interior) of \( (\Omega, k) \) and it is denoted by \( \text{Sint}(\Omega, k) \) (resp. SSint(\( \Omega, k \)), S\text{pint}(\( \Omega, k \)), S\text{bint}(\( \Omega, k \)) and S\text{Shint}(\( \Omega, k \)).

Definition 2.14. [12–14] A S-subset \( (\Omega, k) \) of a S-topological space \( (\mathcal{H}, \tau, k) \) is called:

1. A S-generalized closed set (sg-closed) if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\mathcal{H}, k)) \) and \( (\Psi, k) \) is S-open implies that \( \text{cl}(\Omega, k) \subseteq \text{cl}(\Psi, k) \).
2. A S-semi-generalized closed set (SSg-closed set) if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\Psi, k)) \) and \( (\Psi, k) \) is S-semi-open implies that \( S\text{cl}(\Omega, k) \subseteq \text{cl}(\Psi, k) \).
3. A generalized S-semi-closed set (SGS-closed set) if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\Psi, k)) \) and \( (\Psi, k) \) is S-open implies that \( S\text{cl}(\Omega, k) \subseteq \text{cl}(\Psi, k) \).
4. A S-\( \alpha \)-generalized closed set (S\( \alpha \)-g-closed set) if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\Psi, k)) \) and \( (\Psi, k) \) is S-\( \alpha \)-open implies that \( S\text{cl}(\Omega, k) \subseteq \text{cl}(\Psi, k) \).
5. A S-generalized \( \alpha \)-closed set (S\( \alpha \)-c-closed set) if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\Psi, k)) \) and \( (\Psi, k) \) is S-open implies that \( S\text{cl}(\Omega, k) \subseteq \text{cl}(\Psi, k) \).
6. A S-\( \alpha \)-closed set (S\( \omega \)-closed set) if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\Psi, k)) \) and \( (\Psi, k) \) is S-semi-open implies that \( S\text{cl}(\Omega, k) \subseteq \text{cl}(\Psi, k) \).
7. A S-generalized pre closed set (S\( g \)-closed set) if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\Psi, k)) \) and \( (\Psi, k) \) is S-open implies that \( S\text{cl}(\Omega, k) \subseteq \text{cl}(\Psi, k) \).

Definition 2.15. [15] A map \( \mathcal{U} : (\mathcal{H}, \tau, k) \rightarrow (V, \tau', k') \) is called:

(i) A S-continuous map if \( \mathcal{U}^{-1}(\Omega', k') \) is a S-open set in \( (\mathcal{H}, \tau, k) \), for every S-open set \( (\Omega', k') \) in \( (V, \tau', k') \).
(ii) A S-\( \alpha \)-continuous map if \( \mathcal{U}^{-1}(\Omega', k') \) is a S-\( \alpha \)-open set in \( (\mathcal{H}, \tau, k) \), for every S-open set \( (\Omega', k') \) in \( (V, \tau', k') \).
(iii) A S-semi-continuous map if \( \mathcal{U}^{-1}(\Omega', k') \) is a S-semi-open set in \( (\mathcal{H}, \tau, k) \), for every S-open set \( (\Omega', k') \) in \( (V, \tau', k') \).

3. A Soft generalized \( (\beta, \omega) \)-closed set

Definition 3.1. Let \( (\mathcal{H}, \tau, k) \) be a S-topological space. If \( (\Omega, k) \subseteq (\mathcal{H}, \tau, k) \) and \( (\mathcal{H}, \tau, k) \) is S-\( \omega \)-open set implies that \( S\text{cl}(\Omega, k) \subseteq \text{int}(\mathcal{H}, k) \), then \( (\Omega, k) \) is called a S-generalized \( g(\beta, \omega) \)-closed set. The set of all S-generalized \( g(\beta, \omega) \)-closed sets is denoted by \( Sg(\beta, \omega) \).
(b₁, 〈R〉), (〈Ω₁₄, κ〉) = \{(b₁, 〈R₁₁〉), (b₂, φ)\}, (〈Ω₁₅, κ〉) =
\{(b₁, 〈R₁₂〉), (b₂, 〈R₂₁〉)\}.

**Example 3.1.** Let \(τ = \{〈R, φ, Ω₃, Ω₆, Ω₁₃\}\}. Then, \(τ^c = \{〈R, φ, Ω₁₄, Ω₆, Ω₁₃\}\}. We have that \(g(β, ω) = Ω₁, Ω₂, Ω₄, Ω₅, Ω₇, Ω₉, Ω₁₁, Ω₁₃, Ω₁₄, Ω₁₅\).

**Proposition 3.1.** A set \((Ω, κ)\) is \(S\)-open if and only if \((Ψ, κ)\) is \(S\)-semiclosed and \((Ψ, κ) \subseteq (Ω, κ)\).

**Proof.** Let \((Ω, κ)\) be \(S\)-open. Then \((Ψ', κ)\) is a \(S\)-open set. So, \(cl(Ψ', κ) \subseteq (Ψ, κ)\) whenever \((Ψ', κ) \subseteq (Ψ, κ)\), where \((Ψ, κ)\) is \(S\)-open set. Hence, \((Ψ', κ)\) is \(S\)-semiclosed and so \(int(cl(Ψ', κ)) \subseteq (Ψ, κ)\). If we assume that \(Ψ' = Ψ\), then \(cl(Ψ, κ) \subseteq (Ψ, κ)\). Since \(cl(Ψ', κ) \subseteq (Ψ, κ)\), then \((Ψ, κ)\) is \(S\)-semiclosed. Conversely, let \((Ω, κ)\) be a \(S\)-set such that \((Ψ, κ) \subseteq (Ω, κ)\). Then, \((Ψ', κ) \subseteq (Ω, κ)\). We have that \((Ψ', κ)\) is \(S\)-open set implied that \(cl(Ψ', κ) \subseteq (Ψ, κ)\) then \(cl(Ψ', κ) \subseteq (Ψ, κ)\).

**Proposition 3.2.** Every \(S\)-semiclosed set is a \(Sg(β, ω) - C\) set.

**Proof.** Let \((ψ, κ)\) be a \(S\)-closed set and \((ψ, κ) \subseteq (Ω, κ)\), where \((Ω, κ)\) is \(S\)-open set. Then, by Proposition 3.1 \((ψ, κ) \subseteq (Ω, κ)\). Since \((ψ, κ)\) is \(S\)-closed set, then \((ψ, κ)\) is \(S\)-β-closed set and then \(Sgcl(ψ, κ) = (ψ, κ) \subseteq \int(Ω, κ)\). Hence, \((ψ, κ)\) is \(Sg(β, ω) - C\) set.

We note that the converse of the proposition 3.2 may not be a true, in general as shown in Example 3.1.

**Example 3.2.** Let \((ψ, κ)\) be a \(S\)-closed set and \((ψ, κ) \subseteq (Ω, κ)\), where \((Ω, κ)\) is \(S\)-W-open set. Then, by Lemma 3.1 we have that \((ψ, κ) \subseteq (Ω, κ)\). Since \((ψ, κ)\) is \(S\)-closed, then \((ψ, κ)\) is \(S\)-β-closed and \(Sgcl(ψ, κ) = (ψ, κ) \subseteq \int(Ω, κ)\). Hence, \((ψ, κ)\) is \(Sg(β, ω) - C\) set.

**Example 3.9.** Let \((ψ, κ)\) be a \(S\)-closed set and \((ψ, κ) \subseteq (Ω, κ)\), where \((Ω, κ)\) is \(S\)-W-open set. Then, by Lemma 3.1 we have that \((ψ, κ) \subseteq (Ω, κ)\). Since \((ψ, κ)\) is \(S\)-closed, then \((ψ, κ)\) is \(S\)-β-closed and \(Sgcl(ψ, κ) = (ψ, κ) \subseteq \int(Ω, κ)\). Hence, \((ψ, κ)\) is \(Sg(β, ω) - C\) set.

**Proposition 3.3.** The concept of \(S\)-generalized \((β, ω)\)-closed sets is independent. This is clear from the following example.

**Example 3.9.** Continue to Example 3.1, we have that \(Ω₅\) is \(Sg(β, ω) - C\) set but not \(S\)-α-closed.

**Proposition 3.5.** Arbitrary intersection of \(Sg(β, ω) - C\) sets is also \(Sg(β, ω) - C\) set.

**Proof.** Let \((Ω, κ)\) be \(Sg(β, ω) - C\) sets in the topological space \((〈R, τ, Κ〉)\). Then, \(Sgcl(Ω, κ) \subseteq (Ω, κ)\), for each \(λ\) whenever \(Ω₅, Κ) \subseteq (Ω, κ)\), is \(S\)-β-open sets. Hence, we have that \(n₁Sgcl(Ω, κ) \subseteq (Ω, κ)\), for each \(λ\), whenever \(n₁(Ω, κ) \subseteq (Ω, κ)\). Since \(n₁(Ω, κ)\) is \(S\)-open then \(n₁(Ω, κ)\) is \(Sg(β, ω) - C\) set.

**Remark 3.1.** The S-union of two \(Sg(β, ω) - C\) sets may not be a \(Sg(β, ω) - C\) set. This is clear from the following example.

**Example 3.5.** Continue to Example 3.1, we have that \(Ω₅\) is \(S-β\)-closed but not generalized \((β, ω)\)-closed set.
Remark 3.5. The concept of S-generalized \((\beta, \omega)\)-closed set and S-\(\omega\)-closed set are independent this is clear form the following Example.

Example 3.11. Continue to Example 3.1, we have that \(\Omega_{5}, \Omega_{10}, \Omega_{11}, \Omega_{14}, \Omega_{16}\) are S-generalized \((\beta, \omega)\)-closed set but not S-\(\omega\)-closed set.

Example 3.12. Continue to Example 3.1, let \(\tau = \{\Omega_{1}, \Omega_{2}\}\), \(\tau' = \{\Omega_{1}, \Omega_{2}\}\) we have that, \(\{\Omega_{3}, \Omega_{4}, \Omega_{16}\}\) are S-\(\omega\)-closed set but not S-generalized \((\beta, \omega)\)-closed set.

Remark 3.6. The concept of S-generalized \((\beta, \omega)\)-closed set and S-\(\alpha\)-generalized closed set are independent is clear form the following Example.

Example 3.13. Continue to Example 3.1, we have that \(\Omega_{3}, \Omega_{7}, \Omega_{10}, \Omega_{11}, \Omega_{14}, \Omega_{16}\) are S-generalized \((\beta, \omega)\)-closed set but not S-\(\alpha\)-generalized closed set.

Example 3.14. Continue to Example 3.1, we have that \(\{\Omega_{3}, ..., \Omega_{16}\}\) are S-\(\alpha\)-generalized closed sets but S-generalized \((\beta, \omega)\)-closed sets.

Remark 3.7. The concept of S-generalized \((\beta, \omega)\)-closed set and S-generalized semi-closed set are independent.

Example 3.15. Continue to Example 3.1, we have that \(\Omega_{10}, \Omega_{11}, \Omega_{14}, \Omega_{16}\) are generalized \((\beta, \omega)\)-closed set but not S-generalized semi-closed set.

Example 3.16. Continue to Example 3.1, let \(\tau = \{\Omega_{1}, \Omega_{2}, \Omega_{10}, \Omega_{16}\}\). We have that \(\{\Omega_{3}, \Omega_{4}, \Omega_{5}, \Omega_{6}, \Omega_{7}, \Omega_{8}, \Omega_{9}, \Omega_{10}, \Omega_{11}, \Omega_{12}, \Omega_{13}, \Omega_{14}\}\) are generalized S-semi-closed but not S-generalized \((\beta, \omega)\)-closed.

4. Soft generalized \((\beta, \omega)\)-continuous map

Definition 4.1. A map \(\mathcal{U}: (\mathcal{N}, \tau, \kappa) \to (\mathcal{V}, \tau', \kappa')\) is called a S-generalized \((\beta, \omega)\)-continuous map if \(\mathcal{U}^{-1}(\Omega', \kappa')\) is S-generalized \((\beta, \omega)\)-open set in \((\mathcal{N}, \tau, \kappa)\), for every S-open set \((\Omega', \kappa')\) in \((\mathcal{V}, \tau', \kappa')\).

Example 4.1. Let \(\mathcal{U}: (\mathcal{N}, \tau, \kappa) \to (\mathcal{V}, \tau', \kappa')\) be a map, where \((\mathcal{N}, \tau, \kappa)\) is a topological space defined in Example 3.1 and Let \(\mathcal{V} = \{a, b\}, \kappa' = \{b'_{1}, b'_{2}\}\) and \((\mathcal{V}, \tau', \kappa') = \{V, \phi, \Omega_{1}', \Omega_{2}', \Omega_{3}'\}\) such that \(\Omega_{1}' = \{(b'_{1}, \{a\}), (b'_{2}, \{a\})\}\), \(\Omega_{2}' = \{(b'_{1}, V), (b'_{2}, \{a\})\}\) and \(\Omega_{3}' = \{(b'_{1}, \{a\}), (b'_{2}, V)\}\). Then \(\mathcal{U}\) is a S-generalized \((\beta, \omega)\)-continuous map.

Theorem 4.1. The identity map \(I: (\mathcal{N}, \tau, \kappa) \to (\mathcal{N}, \tau, \kappa)\) is a S-generalized \((\beta, \omega)\)-continuous map.

Proof. It is obvious.

Theorem 4.2. Let \(\mathcal{U}: (\mathcal{R}, \tau, \kappa) \to (\mathcal{V}, \tau', \kappa')\) be a S-map. Then,

1. If \(\mathcal{U}\) is S-continuous, then \(\mathcal{U}\) is S-generalized \((\beta, \omega)\)-continuous.
2. If \(\mathcal{U}\) is S-\(\alpha\)-continuous, then \(\mathcal{U}\) is S-generalized \((\beta, \omega)\)-continuous.
3. If \(\mathcal{U}\) is S-semi-continuous, then \(\mathcal{U}\) is S-generalized \((\beta, \omega)\)-continuous.

Proof. It is obvious.

Conflicts of interest

The authors approve that no conflict of interest.

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