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A.A. Nasef  
*Department of Physics and Engineering Mathematics, Faculty of Engineering, Kafrelsheikh University, Kafrelsheikh (33516), Egypt.*

A.I. Aggour  
*Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City (11884), Cairo, Egypt.*

A. Fathy  
*Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City (11884), Cairo, Egypt.*

S.M. Darwesh  
*Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City (11884), Cairo, Egypt.*

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Soft Topological Notions via Molodtsov Model

Arafa Abdel-zaher Nasefa, Atef Ibrahim Aggour, Ahmed Fathy, Saad Mohamed Darwesh

Abstract

In the present paper, we introduce a new concept of soft sets called soft $g(\beta, \omega)$-closed sets. Also, we study the basic properties of this new concept and we investigate the relation between soft $g(\beta, \omega)$-closed sets and some of the other soft sets. Finally, we introduce the concept of soft $g(\beta, \omega)$-continuous map and we study the relationship between the new concept and some of the other types of soft continuity.

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1. Introduction

Molodtsov [1] introduced the soft set in 1999. The soft sets were employed in application by Maji et al. Furthermore, soft information is a particular information class. Shabir and Naz explored some more fundamental features and introduced the soft topological space in [2]. Following that, some topological research discovered several fresh varieties of near soft open sets and investigated both their individual and interrelated characteristics. K. Kannan first discussed the idea of a soft generalised closed set in [3]. Many different soft generalised closed set types were then defined by various topologists. In this article, we introduced a brand-new class of soft generalised sets termed soft $(\beta, \omega)$-closed and described its fundamental characteristics. Recent years have seen a significant growth in the number of articles regarding soft sets and their applications in numerous disciplines, as demonstrated in [4–6].

2. Preliminaries

In this section, we present the basic definition and some results of soft set theory. Let $\mathcal{R}$ be a universal set and $\kappa$ be the set of parameters, $P(\mathcal{R})$ is the power set of $\mathcal{R}$; $A \subseteq \kappa$ and the soft set will be denoted by $S$-set.

Definition 2.1 [1]. A S-set $(\Omega, A)$ on $\mathcal{R}$ is defined by the set of ordered pairs $(\Omega, A) = \{ (h, \Omega_A(h)) : h \in \kappa, \Omega_A(h) \in P(V) \}$, where $\Omega_A : A \rightarrow P(\mathcal{R})$.

Definition 2.2 [1,7]. A S-set $(\Omega, A)$ is called null S-set if for all, $h \in A$, then $\Omega(h) = \phi$ and its denoted by $\phi$. A S-set $(\Omega, A)$ is called absolute S-set if for all $a \in A, \Omega(h) = R$ and its denoted by $\mathcal{R}$.

Definition 2.3 [1,7]. Let $(\Omega, A)$ and $(\Psi, \beta)$ be two S-sets over $\mathcal{R}$. Then, the union of $(\Omega, A)$ and $(\Psi, \beta)$ is a S-set $(H, C)$ where $C = (A \cup \beta)$ and $H(h) = \Omega(h)$ if $h \in A - \beta$, $H(h) = \Psi(h)$ if $h \in \beta - A$, $H(h) = \Omega(h) \cup \Psi(h)$ if $h \in A \cap \beta$.

Definition 2.4 [1,7]. Let $(\Omega, A)$ and $(\Psi, \beta)$ be two S-sets over $\mathcal{R}$. The intersection of $(\Omega, A)$ and $(\Psi, \beta)$ is a S-set $(F, D)$ where $D = A \cap \beta$, $F(h) = \Omega(h) \cap \Psi(h)$, for all $h \in D$.

Definition 2.5 [1,7]. A S-set $(\Omega, A)$ is called a S-subset of $(\Psi, \beta)$ if $A \subseteq \beta$ and $(\Omega(h) \subseteq \Psi(h)$ for all $a \in A$. We write $(\Omega, A) \subseteq (\Psi, \beta)$.

Definition 2.6 [1,7]. Let $(\Omega, \kappa), (\Psi, \kappa)$ be two S-sets over $\mathcal{R}$. Then, the difference of $(\Omega, \kappa), (\Psi, \kappa)$ is denoted by $(H, C) = (\Omega, \kappa) \setminus (\Psi, \kappa)$ such that $H(C) = \Omega(h) \setminus \Psi(h)$ for all $h$ in $A$.

Definition 2.7 [1,7]. The relative complement of $(\Omega, A)$ is denoted by $(\Omega, A)^c = (\Omega, A)$ where $\Omega^c : A \rightarrow P(\mathcal{R})$, such that $\Omega^c(h) = R \setminus \Omega(h)$ for all $h$ in $A$. 

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* Corresponding author.
E-mail address: ahmedfathy@azhar.edu.eg (A.A.-z. Nasef).
Definition 2.8 [2,8]. Let τ be a collection of S-sets over ℳ. Then, τ is called a S-topology on ℳ if the following axioms are satisfied:

1. \( \emptyset, \mathcal{M} \in \tau \).
2. The union of arbitrary S-sets in \( \tau \) belongs to \( \tau \).
3. The intersection of two S-sets in \( \tau \) belongs to \( \tau \).

The triple \( (\mathcal{M}, \tau, k) \) is called a S-topological space and the members of \( \tau \) are called S-open sets and its complement are called S-closed sets.

Definition 2.9. [2,8] The S-interior of \( (\Omega, k) \) is the union of all S-open sets of topological space \( (\mathcal{M}, \tau, k) \) contained in \( (\Omega, k) \) and its denoted by int(\( \Omega, k \)).

Definition 2.10. [2,8] The S-closure of \( (\Omega, k) \) is the intersection of all S-closed sets containing and its denoted by cl(\( \Omega, k \)).

Definition 2.11. [6,9–11] Let \( (\mathcal{M}, \tau, k) \) be a S-topological space. Then, \( (\Omega, k) \) is said to be:

1. A S-α-open set if \( (\Omega, k) \subseteq \text{int}(\text{cl}(\text{int}(\Omega, k))) \).
2. A S-semi-open set if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\Omega, k)) \).
3. A S-pre open set if \( (\Omega, k) \subseteq \text{cl}(\text{int}(\Omega, k)) \).
4. A S-b-open set if \( (\Omega, k) \subseteq \text{cl}(\text{cl}(\text{int}(\Omega, k))) \).
5. A S-β-open set if \( (\Omega, k) \subseteq \text{cl}(\text{cl}(\text{int}(\Omega, k))) \).

The family of all S-α-open (resp. S-semi-open, S-pre open, S b-open and S-β-open) sets in a S-topological space \( (\mathcal{M}, \tau, k) \), is denoted by SaO (resp. SSO, SPO, SbO and SSO).

Definition 2.12. [6,9–11] A set \( (\Omega, k) \) of a S-topological space \( (\mathcal{M}, \tau, k) \) is called S-α-closed (resp. S-semi-closed, S-pre closed, S b-closed and S-β-closed) sets if its complements is S-α-open (resp. S-semi-open, S-pre open, S b-open and S-β-open) sets.

Definition 2.13. [6,9–11] Let \( (\mathcal{M}, \tau, k) \) be a S-topological space and \( (\Omega, k) \) be a S-set. Then, the intersection of all S-α-closed (resp. S-semi-closed, S-pre closed, S b-closed and S-β-closed) sets containing \( (\Omega, k) \) is called S-α-closure (resp. S-semi-closure, S-pre closure, S b-closure and S-β-closure) of \( (\Omega, k) \) and its denoted by S\text{cl}(\( \Omega, k \)) (resp. S\text{sc}(\( \Omega, k \)), S\text{pre}(\( \Omega, k \)), S\text{bcl}(\( \Omega, k \)), S\text{bcl}(\( \Omega, k \)).

(1) S\text{cl}(\( \Omega, k \)) and S\text{sc}(\( \Omega, k \)).
(2) The union of all S-α-open (resp. S-semi-open, S-pre open, S b-open and S-β-open) sets containing in \( (\Omega, k) \) is called S-α-interior (resp. S-semi-interior, S-pre interior, S b-interior and S-β-interior) of \( (\Omega, k) \) and it is denoted by S\text{int}(\( \Omega, k \)) (resp. S\text{semi}(\( \Omega, k \)), S\text{pre}(\( \Omega, k \)), S\text{b}(\( \Omega, k \)), S\text{bint}(\( \Omega, k \))).

Definition 2.14. [12–14] A S-subset \( (\Omega, k) \) of a S-topological space \( (\mathcal{M}, \tau, k) \) is called:

1. A S-generalized closed set (sg-closed) if \( (\Omega, k) \subseteq (\Psi, k) \) and \( (\Psi, k) \) is S-open implies that \( \text{cl}(\Omega, k) \subseteq (\Psi, k) \).
2. A S-semi-generalized closed set (SSg-closed set) if \( (\Omega, k) \subseteq (\Psi, k) \) and \( (\Psi, k) \) is S-semi-open implies that \( \text{Scl}(\Omega, k) \subseteq (\Psi, k) \).
3. A generalized S-semi-closed set (SGs-closed set) if \( (\Omega, k) \subseteq (\Psi, k) \) and \( (\Psi, k) \) is S-open implies that \( \text{Scl}(\Omega, k) \subseteq (\Psi, k) \).
4. A S-α-generalized closed set (Sα g-closed) if \( (\Omega, k) \subseteq (\Psi, k) \) and \( (\Psi, k) \) is S-α-open implies that \( \text{Scl}(\Omega, k) \subseteq (\Psi, k) \).
5. A S-generalized α-closed set (SG α-closed set) if \( (\Omega, k) \subseteq (\Psi, k) \) and \( (\Psi, k) \) is S-open implies that \( \text{Sacl}(\Omega, k) \subseteq (\Psi, k) \).
6. A S-α-closed set (Sα c-closed set) if \( (\Omega, k) \subseteq (\Psi, k) \) and \( (\Psi, k) \) is S-semi-open implies that \( \text{Sacl}(\Omega, k) \subseteq (\Psi, k) \).
7. A S-generalized pre closed set (SGp-closed set) if \( (\Omega, k) \subseteq (\Psi, k) \) and \( (\Psi, k) \) is S-open set implies that \( \text{Spcl}(\Omega, k) \subseteq (\Psi, k) \).

Definition 2.15. [15] A map \( \Phi : (\mathcal{M}, \tau, k) \) (\( V, \tau', k' \)) is called:

(i) A S-continuous map if \( \Phi^{-1}(\Omega, k') \) is a S-open set in \( (\mathcal{M}, \tau, k) \), for every S-open set \( (\Omega, k') \) in \( (\mathcal{V}, \tau', k') \).
(ii) A S-α- continuous map if \( \Phi^{-1}(\Omega, k') \) is a S-α-open set in \( (\mathcal{M}, \tau, k) \), for every S-open set \( (\Omega, k') \) in \( (\mathcal{V}, \tau', k') \).
(iii) A S-α-closed map if \( \Phi^{-1}(\Omega, k') \) is an S-α-closed set in \( (\mathcal{M}, \tau, k) \), for every S-open set \( (\Omega, k') \) in \( (\mathcal{V}, \tau', k') \).

3. A Soft generalized \((\beta, \omega)\)-closed set

Definition 3.1. Let \( (\mathcal{M}, \tau, k) \) be a S-topological space. If \( (\Omega, k) \subseteq (\mathcal{M}, \tau, k) \) and \( (\Psi, k) \) is S-ω-open set implies that \( \text{bl}(\Omega, k) \subseteq \text{int}(\Psi, k) \), then \( (\Omega, k) \) is called a S-generalized \( g(\beta, \omega) \) closed set. The set of all S-generalized \( g(\beta, \omega) \) closed sets is denoted by \( Sg(\beta, \omega) \).

In this paper, we consider \( \mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2\} \) and \( \{h_1, h_2\}, (\Omega, k) = \mathcal{M} = \{h_1, \mathcal{M}_2\} \), \( (\Omega_2, k) = \phi \), \( (\Omega_4, k) = \{h_1, \mathcal{M}_2\} \), \( (\Omega_8, k) = \{h_1, \mathcal{M}_2\} \), \( (\Omega_5, k) = \{h_1, \mathcal{M}_2\} \), \( (\Omega_6, \phi) = \{h_1, \mathcal{M}_2\} \), \( (\Omega_7, k) = \{h_1, \mathcal{M}_2\} \), \( (\Omega_9, k) = \{h_1, \mathcal{M}_2\} \), \( (\Omega_{10}, k) = \{h_1, \mathcal{M}_2\} \).
Definition 3.1. A set \( (\Omega, k) \) is \( S \)-open if and only if for \( (\Psi, k) \subseteq \Omega \), \( \text{int}(\Omega, k) \subseteq (\Psi, k) \). Hence, \( (\Psi, k) \) is \( S \)-semi-open and so \( \text{int}(\text{cl}(\Psi, k)) \subseteq (\Psi, k) \).

Example 3.2. Let \( \Omega = (\mathbb{R}, \mathbb{R}) \), then \( \mathbb{R}^2 = (\mathbb{R}, \mathbb{R}) \).

Proposition 3.1. Every \( S \)-semi-open set is \( S \)-closed set.

Example 3.3. Let \( \tau = \{ (\mathbb{R}, \mathbb{R}), \mathbb{R}, \mathbb{R}, \mathbb{R} \} \). Then, \( \mathbb{R}^2 = (\mathbb{R}, \mathbb{R}) \).

Proposition 3.2. Every \( S \)-semi-closed set is \( S \)-open set.

Example 3.4. Let \( (\mathbb{R}, k) \) be \( S \)-open set.

Proposition 3.3. Arbitrary intersection of \( S \)-open sets is also \( S \)-open set.

Example 3.5. Continue to Example 3.1, we have that \( \Omega_2 \Omega_6 \Omega_16 \) are \( S \)-\( \omega \)-closed sets.

Example 3.6. Continue to Example 3.1, we have that \( \Omega_2 \mathbb{R}_7 \mathbb{R}_16 \mathbb{R}_11 \mathbb{R}_13 \mathbb{R}_14 \mathbb{R}_16 \) are \( S \)-\( \omega \)-closed sets but not \( S \)-\( \omega \)-closed sets.

Example 3.7. Continue to Example 3.1, let \( \tau = \{ \Omega_1, \Omega_2, \Omega_6 \} \). Then, we have that \( \mathbb{R}^2 = (\mathbb{R}, \mathbb{R}) \).

Example 3.8. Continue to Example 3.1, we have that \( (\mathbb{R}, k) \) is \( S \)-\( \omega \)-closed set and \( \text{int}(\Omega, k) \subseteq (\Psi, k) \).

Example 3.9. Continue to Example 3.1, we have that \( \Omega_2 \mathbb{R}_7 \mathbb{R}_16 \mathbb{R}_11 \mathbb{R}_13 \mathbb{R}_14 \mathbb{R}_16 \) are \( S \)-\( \omega \)-generalized \( (\beta, \omega) \)-closed sets and \( \text{int}(\Omega, k) \subseteq (\Psi, k) \).

Example 3.10. Continue to Example 3.1, we have that \( \Omega_2 \mathbb{R}_7 \mathbb{R}_16 \mathbb{R}_11 \mathbb{R}_13 \mathbb{R}_14 \mathbb{R}_16 \mathbb{R}_19 \mathbb{R}_11 \mathbb{R}_13 \mathbb{R}_14 \mathbb{R}_15 \mathbb{R}_16 \) are \( S \)-\( \omega \)-generalized \( (\beta, \omega) \)-closed sets but not \( S \)-\( \omega \)-generalized \( (\beta, \omega) \)-closed sets.

Example 3.11. Continue to Example 3.1, we have that \( \Omega_2 \mathbb{R}_7 \mathbb{R}_16 \mathbb{R}_11 \mathbb{R}_13 \mathbb{R}_14 \mathbb{R}_16 \mathbb{R}_19 \mathbb{R}_11 \mathbb{R}_13 \mathbb{R}_14 \mathbb{R}_15 \mathbb{R}_16 \) are \( S \)-\( \omega \)-generalized \( (\beta, \omega) \)-closed sets but not \( S \)-\( \omega \)-generalized \( (\beta, \omega) \)-closed sets.
Remark 3.5. The concept of $S$-generalized $(\beta, \omega)$-closed set and $S$-$\omega$-closed set are independent this is clear from the following Example.

Example 3.11. Continue to Example 3.1, we have that $\Omega_5, \Omega_{10}, \Omega_{11}, \Omega_{14}, \Omega_{16}$ are $S$-generalized $(\beta, \omega)$-closed set but not $S$-$\omega$-closed set.

Example 3.12. Continue to Example 3.1, let $\tau = \{\Omega_1, \Omega_2\}, \tau' = \{\Omega_1, \Omega_2\}$ we have that, $\{\Omega_3, \Omega_4, \Omega_{16}\}$ are $S$-$\omega$-closed set but not $S$-generalized $(\beta, \omega)$-closed set.

Remark 3.6. The concept of $S$-generalized $(\beta, \omega)$-closed set and $S$-$\alpha$-generalized closed set are independent is clear form the following Example.

Example 3.13. Continue to Example 3.1, we have that $\Omega_5, \Omega_7, \Omega_{10}, \Omega_{11}, \Omega_{13}, \Omega_{14}, \Omega_{16}$ are $S$-generalized $(\beta, \omega)$-closed set but not $S$-$\alpha$-generalized closed set.

Example 3.14. Continue to Example 3.1, we have that $\{\Omega_3, \ldots, \Omega_{16}\}$ are $S$-$\alpha$-generalized closed sets but $S$-generalized $(\beta, \omega)$-closed sets.

Remark 3.7. The concept of $S$-generalized $(\beta, \omega)$-closed set and $S$-generalized semi-closed set are independent.

Example 3.15. Continue to Example 3.1, we have that $\Omega_5, \Omega_{10}, \Omega_{11}, \Omega_{14}, \Omega_{16}$ are generalized $(\beta, \omega)$-closed set but not $S$-generalized semi-closed set.

Example 3.16. Continue to Example 3.1, let $\{\tau = \{\Omega_1, \Omega_2, \Omega_{10}, \Omega_{16}\}\}$. We have that $\{\Omega_3, \Omega_4, \Omega_5, \Omega_6, \Omega_7, \Omega_8, \Omega_9, \Omega_{10}, \Omega_{11}, \Omega_{12}, \Omega_{13}, \Omega_{14}\}$ are generalized S-semi-closed but not $S$-generalized $(\beta, \omega)$-closed.

4. Soft generalized $(\beta, \omega)$-continuous map

Definition 4.1. A map $U : (\mathcal{R}, \tau, \kappa) (V, \tau', \kappa')$ is called a $S$-generalized $(\beta, \omega)$-continuous map if $U^{-1}(\Omega', \kappa')$ is $S$-generalized $(\beta, \omega)$-open set in $(\mathcal{R}, \tau, \kappa)$ for every S-open set $(\Omega', \kappa')$ in $(V, \tau', \kappa')$.

Example 4.1. Let $U : (\mathcal{R}, \tau, \kappa) (V, \tau', \kappa')$ be a map, where $(\mathcal{R}, \tau, \kappa)$ is a topological space defined in Example 3.1 and Let $V = \{a, b\}, \kappa' = \{b_1, b_2\}$, and $(V, \tau', \kappa') = \{a, b, \Omega_1', \Omega_2', \Omega_3\}$ such that $\Omega_1' = \{b_1, \{a\}\}, b_2, \{a\}\}, \Omega_2' = \{b_1, V\}, b_2, \{a\}\}$ and $\Omega_3' = \{b_1, \{a\}\}, b_2, V\}$. Then $U$ is a $S$-generalized $(\beta, \omega)$-continuous map.

Theorem 4.1. The identity map $I : (\mathcal{R}, \tau, \kappa) \rightarrow (\mathcal{R}, \tau, \kappa)$ is a S-generalized $(\beta, \omega)$-continuous map.

Proof. It is obvious.

Theorem 4.2. Let $U : (\mathcal{R}, \tau, \kappa) (V, \tau', \kappa')$ be a $S$-map. Then,

(1) If $U$ is S-continuous, then $U$ is S-generalized $(\beta, \omega)$-continuous.

(2) If $U$ is S-$\alpha$-continuous, then $U$ is S-generalized $(\beta, \omega)$-continuous.

(3) If $U$ is S-semi-continuous, then $U$ is S-generalized $(\beta, \omega)$-continuous.

Proof. It is obvious.

Conflicts of interest

The authors approve that no conflict of interest.

References