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VISIBILITY INTERVALS BETWEEN TWO SATELLITES ORBITS UNDER J_2-GRAVITY AND DRAG FORCE

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ABSTRACT

The research introduces a general analytical and computational technique for satellite-to-satellite visibility. The effect of earth's oblateness and drag force were taken into account.The Visibility function in terms of the orbital elements of the two satellites and the time were derived. The rise and set periods of the satellites are determined through the sign of the visibility function. Numerical examples were worked for some satellites.

Keywords: Visibility function – line of site- - rise and set times-Earth oblateness- Drag force.

1. INTRODUCTION

The rise/set problem may be defined as the process of determining the times at which a satellite rises and sets with respect to a ground location. The easiest solution uses a numerical method to determine visibility periods for the site and satellite by evaluating UK position vectors of each. It advances vectors by a small time increment, Δt, and checks visibility at each step. A disadvantage of this method is the computation time, especially when modeling many perturbations and processing several satellites. Escobal [1], [2] proposed a faster method to solve the rise/set problem by developing a closed-form solution for unrestricted visibility periods about an oblate Earth. He assumes infinite range, azimuth, and elevation visibility for the site.

Escobal transforms the geometry for the satellite and tracking station into a single transcendental equation for time as a function of eccentric anomaly. He then uses numerical methods to find the rise and set anomalies, if they exist. Lawton[3] has developed another method to solve for satellite-satellite and satellite-ground station visibility periods for vehicles in circular or near circular orbits by approximating the visibility function, by a Fourier series. More recently, Alfano, Negron, and Moore[4]derived an analytical method to obtain rise/set times of a satellite for a ground station and includes restrictions for range, azimuth, and elevation. The algorithm uses pairs of fourth-order polynomials to construct functions that represent the restricted parameters (range, azimuth, and elevation) versus time for an oblate Earth. It can produce these functions from either uniform or arbitrarily spaced data points. The viewing times are obtained by extracting the real roots of localized quantic.

Palmar [5] , introduced a new method to predict the passes of satellite to a specific target on the ground which is useful for solving the satellite visibility problem. He firstly described a coarse search phase of this method including two-body motion, secular perturbation and atmospheric drag, then he described the second phase (refinement), which uses a further developed controlling equation F(α) = 0 based on the epicycle equations.

In the present work, a fast method for satellite-satellite visibility periods for the rise-and-set time prediction for two satellites in terms of classical orbital elements of the two satellites and time were established. The secular variations of the orbital elements due to Earth oblateness and drag force were taken into account in order to consider the changes in the nodal period of satellite and the changes in the long term prediction of maximum elevation angle. In the following description the formulae for satellite rise-and-set times of the two satellites were introduced. The derived visibility function provides high accuracy over a long period.

2. VISIBILITY ANALYSIS

The location of a satellite is determined by the

Kepler's laws. A set of six orbital parameters[*a, e, i, Ω,ω,f*] is used to fully describe the position of a satellite in a point in space at any given time: semi-major axis a, eccentricity e, inclination of the orbit plane i, right ascension of the node Ω, the argument of perigee ω, and true anomaly f. The above parameters are shown in the **Figure1**.

The links between two satellites are determined by the visibility analysis presented as follows:

Referring to Fig. 1, the position vectors of satellites 1, and 2 with respect to the ECI coordinate system are $\underline{r}_1$ and $\underline{r}_2$. The position vector from satellite 1 to satellite 2 will be denoted by $\vec{\rho} = \vec{r}_2 - \vec{r}_1$

Let $h = OP = a_e + \Delta$, be the perpendicular from the dynamical center of the earth *to the range vector* $\vec{\rho}$*, cutting the earth surface at Q and the range vector at P , where* a_e *is the mean radius of the Earth, and Δ is the thickness of the atmosphere above the surface of the Earth to P. From the geometry of Fig.1, we have the following relations:*

$$\text{Area Of Triangle }_1S_2 = \frac{1}{2}|\vec{r}_1 \wedge \vec{r}_2| = \frac{1}{2}|\vec{r}_1||\vec{r}_2|\sin\Psi \quad (2.1$$

On the other hand, the area can be calculated from the relation:

$$\text{Area of Triangle } OS_1 S_2 = \frac{1}{2}|\vec{r}_1 - \vec{r}_2| \cdot h \quad (2.2)$$

From (1) and (2), we conclude that:

$$h = a_e + \Delta = \frac{|\vec{r}_1||\vec{r}_2|\sin\Psi}{|\vec{r}_1 - \vec{r}_2|} \qquad (2.3)$$

where ψ, is the angle between $\underline{r}_1$ and $\underline{r}_2$ of the two satellites.

The condition for direct visibility between the two satellites can be determined from (3) by putting

$$\Delta = \frac{|\vec{r}_1||\vec{r}_2|\sin\Psi}{|\vec{r}_1 - \vec{r}_2|} - a_e$$

where

$$\Psi = \cos^{-1}\left(\frac{\vec{r}_1 \cdot \vec{r}_2}{|\vec{r}_1| \cdot |\vec{r}_2|}\right) \qquad (2.4)$$

After the determination of the locations of satellites at any time in the space, they can only achieve visibility when they are both above the tangent plane to the earthsurface. The extreme situation is that both of them are in the tangent plane.

When $\Delta > 0$, the two satellites can achieve visibility. Otherwise, there is no visibility.

where

$$D = \sqrt{\frac{r_1^2 r_2^2 - (\vec{r}_1 \cdot \vec{r}_2)^2}{r_1^2 + r_2^2 - (\vec{r}_1 \cdot \vec{r}_2)}} \qquad (2.5\ )$$

Unless there's an impact between the two satellites, the dominator will not equal zero. The sign of **Δ** associated with visibility can be obtained as:

Positive value of $\Delta \Rightarrow$ Direct line-of-sight

Negative value of $\Delta \Rightarrow$ non-visibility

3.Coordinate Transformation.

The coordinate transformation presented here is the passage from the peri-focal coordinate system (P Q W) with unit vectors $\vec{P}$ *and* $\vec{Q}$ to the geocentric–equatorial system (X Y Z) with unit vectors (I J K). Here $\vec{P}$and $\vec{Q}$ are the unit vector sin the orbital plane of the satellite, where $\vec{P}$ originated from focus towards the perigee of the orbit and$\vec{Q}$is advanced to $\vec{P}$ by a right angle in the direction of motion. In the peri-focal coordinate system, we have:

$$(\vec{r}_i)_{PQW} = \begin{bmatrix} \xi_i \\ \eta_i \\ 0 \end{bmatrix}, \qquad i = 1,2 \qquad (3.1)$$

where $\xi_i = r_i \cos f_i$, $\eta_i = r_i \sin f_i$, and

$$r_i = \frac{p_i}{1 + e_i \cos f_i}, \ and\ \ p_i = a_i(1 - e_i^2),$$

p_i is the semi-latus-rectum.

In order to obtain the position vectors in the geocentric equatorial coordinates, we rotate the peri-focal coordinate system through the following rotations (Vallado,2007)[6]:

$$\vec{r}_{IJK} = [ROT3(-\Omega)][ROT1(-i)][ROT3(-\omega)]$$
(3.2)

So that the transformation matrix will take the form:

$$\left[\frac{IJK}{PQW}\right]=\begin{bmatrix}\cos\Omega\cos\omega-\sin\Omega\sin\omega\cos I & -\cos\Omega\sin\omega-\sin\Omega s\cos\omega\cos I & \sin\Omega\sin I\\ \sin\Omega\cos\omega+\cos\Omega\sin\omega\cos I & -\sin\Omega\sin\omega+\cos\Omega\cos\omega\cos I & -\cos\Omega\sin I\\ \sin\omega\sin I & \cos\omega\sin I & \cos I\end{bmatrix}$$

(3.3)

Now, we can write

$$\vec{r}_1\cdot\vec{r}_2 = A_1\xi_1\xi_2 + A_2\eta_1\xi_2 + A_3\eta_2\xi_1 + A_4\eta_1\eta_2$$

, (3.4)

where

$$A_2=\vec{Q}_1\cdot\vec{P}_2,\ ,\ A_4=\vec{Q}_1\cdot\vec{Q}_2 \qquad (3.5)$$

Substituting Equations (3.5), using the elements of the transformation Equation (3.3) into Equation (3.4) we obtain

$$\vec{r}_1\cdot\vec{r}_2=\frac{p_1p_2\{D_1\cos f_1\cos(\gamma_1-f_1)+D_2\cos f_2\cos(\Psi_1-f_2)\}}{[1+e_1\cos f_1][1+e_2\cos f_2]} \qquad (3.6)$$

where

$$\sin\Psi_1=\frac{A_2}{\sqrt{A_1+A_2}}\quad and\quad \cos\Psi_1=\frac{A_1}{\sqrt{A_1+A_2}}$$

$$\sin\gamma_1=\frac{A_3}{\sqrt{A_3+A_4}}\quad and\quad \cos\gamma_1=\frac{A_4}{\sqrt{A_3+A_4}}$$

$$D_1=\frac{\sqrt{A_2}}{\sqrt{A_1+A_2}}$$

Hence, the condition of visibility in Eq. (2.5) can be written in the form

$$\Delta=\left[\frac{p_1^2p_2^2(1-\Pi^2)}{p_1^2[1+e_1\cos f_1]+p_2^2[1+e_2\cos f_2]-2\Pi p_1p_2[1+e_1\cos f_1][1+e_2\cos f_2]}\right]^{\frac{1}{2}}-a_e$$

(3.7)

where

$$\Pi=D_1\cos(\gamma_1-f_1)\cos(f_2)+D_1\cos(\Psi_1-f_1)\sin(f_2)$$

4. Perturbing Forces

Here we shall consider the effect of perturbation on the orbital elements due to both the Earth oblateness and Atmospheric drag. So, we will express the orbital elements of the two satellites in the form

$$\sigma_i(t)=\sigma_{i0}+(\Delta\sigma_i)_{obl}+(\Delta\sigma_i)_D,$$

Where $(\Delta\sigma_i)_{obl}$ *and* $(\Delta\sigma_i)_D$ denote the perturbations in the elements due to oblateness, and drag force respectively.

4.1. The effect of Earth Oblateness

Analytical investigation of the oblateness effects of a central body on a satellite has shown that certain elements, such as M,Ω,ω experience secularvariations (increasing or decreasing) from the adopted epoch values and periodic variations about these epoch values. Other elements such as a, i and e retain only periodic variations, a further distinction is made between short period variation and long period variations.

The oblateness variations are caused by the continuous variance of ω, owing to the fact that the trigonometric functions of ω have secular variations with period 2π (Escobal,1965)[2].

The gravitational potential, U, of a satellite including the contribution of J_2, is given by(Satellites)[7]

$$\Phi=\frac{\mu}{r}\left[1+\frac{J_2R_e^2}{2r^2}\left(1-3\sin^2\delta\right)\right] \qquad (4.1)$$

which may be expressed as:

$$\Phi=V_0+R \qquad (4.2)$$

where

V_0: is the gravitational potential of purely spherical Earth;

R: the disturbing potential due to the Earth oblateness;

$M = k^2 m$: the gravitational constant × mass of the Earth,

J_2: Coefficient of the 2nd zonal harmonic.

δ: the satellite altitude.

Using the relation

$$\sin(\delta)=\sin(i)\sin(f+\omega)$$

The perturbing function takes the following form

$$R = \frac{k^2 m}{r}\frac{3}{2}\frac{J_2}{a^3}\left(\frac{a}{r}\right)^3\left\{\frac{1}{3}-\frac{1}{2}\sin^2 I+\frac{1}{2}\sin^2 I\cos(f+\omega)\right\} \tag{4.3}$$

Since we are interested in the secular variation, so we eliminate the short and long period terms from the perturbing function. So, to average *R*, we integrate (4.3) with respect to M between zero and 2π . The secular part of the perturbing function can be obtained as:

$$\widetilde{R} = k^2 m\left[\frac{3}{2}\frac{J_2}{a^3}\frac{R_e^2}{(1-e^2)^{\frac{3}{2}}}\left\{\frac{1}{3}-\frac{1}{2}\sin^2 i\right\}\right] \tag{4.4}$$

In order to account for the secular variation of the elements, we use the equations of variations of parameters (VOP) in the Lagrange's form (Escobal,1965)[2]:

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial\widetilde{R}}{\partial M}$$

$$\frac{de}{dt} = \frac{1-e^2}{na^2 e}\frac{\partial\widetilde{R}}{\partial M} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial\widetilde{R}}{\partial\omega}$$

$$\frac{d\omega}{dt} = -\frac{\cos i}{na^2\sin I\sqrt{1-e^2}}\frac{\partial\widetilde{R}}{\partial i} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial\widetilde{R}}{\partial e}$$

$$\frac{di}{dt} = -\frac{\cos i}{na^2\sin i\sqrt{1-e^2}}\frac{\partial\widetilde{R}}{\partial\omega} \tag{4.5}$$

$$\frac{d\Omega}{dt} = -\frac{1}{na^2\sin I\sqrt{1-e^2}}\frac{\partial\widetilde{R}}{\partial i}$$

$$\frac{dM}{dt} = n - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial\widetilde{R}}{\partial e} - \frac{2}{na}\frac{\partial\widetilde{R}}{\partial a}$$

Substitution of (22) into (23)) yield

$$\frac{d\bar{a}}{dt} = 0, \quad \frac{d\bar{e}}{dt} = 0, \quad \frac{d\bar{i}}{dt} = 0,$$

$$\frac{d\overline{\Omega}}{dt} = -\frac{3}{2(1-e^2)^2}\bar{n}J_2\left(\frac{R_e}{a}\right)^2\cos i$$

$$\frac{d\overline{\omega}}{dt} = \frac{3}{4(1-e^2)^2}\bar{n}J_2\left(\frac{R_e}{a}\right)^2(5\cos^2 i-1)$$

$$\frac{d\overline{M}}{dt} - \bar{n} = \Delta n = \frac{3}{4(1-e^2)^{\frac{3}{2}}}\bar{n}J_2\left(\frac{R_e}{a}\right)^2(3\cos^2 i-1) \tag{4.6}$$

where $\bar{n} = \sqrt{\mu/a^3}$

To calculate the variations in the parameters which experience the secular variations, we find that

$$\frac{dM}{dt} = n\left[1+\frac{3}{2}J_2\frac{\sqrt{1-e^2}}{p^2}^2\left\{1-\frac{3}{2}\sin^2 i\right\}\right] \equiv \bar{n}$$

$$\Omega = \Omega_0 - \left(\frac{3}{2}\frac{J_2}{P^2}\cos i\right)n_0(t-t_0) \tag{4.7}$$

$$\omega = \omega_0 + \frac{3}{2}\frac{J_2}{P^2}\left(2-\frac{5}{2}\sin^2 i\right)n_0(t-t_0) \tag{4.8}$$

$$M = M_0 + \left[1+\frac{3}{2}J_2\frac{\sqrt{1-e^2}}{P^2}\left(1-\frac{3}{2}\sin^2 i\right)\right]n_0(t-t_0) \tag{4.9}$$

4.2. Effects of Drag

The predominant effect of drag is to shrink the orbit and in many cases, cause the satellite to reenter the atmosphere and hit the Earth. For satellites that are close to the Earth, the possibility of reentry is important in determining a satellite's lifetime.

Drag is a non-conservative force that acts to lower a satellite's orbit (less than 600 km). A satellite experiences varying effects from drag depending on its attitude . A strong wind is analogous to the density variations in the atmosphere (diurnal, solar, and geomagnetic for example).

Drag force is given by(Roy, 2005)[8].

$$\overline{F}_D = -\frac{1}{2}B_c\rho|V_S|\hat{V}_S, \tag{4.10}$$

Where

$B_c = (A/m)C_D$ is the ballistic coefficient

C_D is the aerodynamic drag coefficient,

A/m is the satellite area to mass ratio,

ρ is the air density,

V_S is the velocity of the satellite relative to the atmosphere

$\hat{V}_S$ is the unit vector in the satellite velocity direction

Since the drag is non-conservative force, so we will use variation of parameters equations (VOP) in the Gaussian form , given in the RSW system(Roy, 1988)[9]by:

$$\frac{da}{dt}=\frac{2}{n\sqrt{1-e^2}}(e\sin f)F_R+(1+e\cos f)F_S$$

$$\frac{de}{dt}=\frac{\sqrt{1-e^2}}{na}\left[(\sin f)F_R+\frac{(e+2\cos f+e\cos^2 f)}{(1+e\cos f)}F_S\right]$$

$$\frac{di}{dt}=\frac{\sqrt{1-e^2}}{na}\frac{\cos(f+\omega)}{(1+e\cos f)}F_W$$

$$\frac{d\Omega}{dt}=\frac{\sqrt{1-e^2}}{na\sin i}\frac{\sin(f+\omega)}{(1+e\cos f)}F_W$$

$$\frac{d\omega}{dt}=\frac{\sqrt{1-e^2}}{nae}\left[-(\cos f)F_R+\frac{(e+2\cos f)}{(1+e\cos f)}(\sin f)F_S-\frac{e\sin(f+\omega)}{(1+e\cos f)}(\cot i)F_W\right]$$

$$\frac{dM}{dt}=n+\frac{1-e^2}{nae}\left[\left(\sin f-\frac{2e}{(1+e\cos f)}\right)F_R-\frac{(e+2\cos f)}{(1+e\cos f)}(\sin f)F_S\right] \quad (4.11)$$

Where

$$\vec{F}_D=F_R\widehat{R}+F_S\widehat{S}+F_W\widehat{W} \quad (4.12)$$

and $\underline{R},\underline{S}, and\,\underline{W}$ are respectively, the unit vectors in the radial direction, transverse direction, and normal to the orbital plane of the satellite shown on Fig.2.Thus in these coordinates, the drag force components will be:

$$\vec{F}_D=\left[-|F_D|\cos\varphi,-|F_D|\sin\varphi.0\right]$$

where φ is the flight path angle .

Since the drag force oppose the velocity vector. Hence, we need to find the drag components in the TNW – coordinate system, where T- axis aligned along the tangent (velocity vector) , N – axis normal to it in the direction of increasing the true anomaly, f , and W – axis completes the triad in the positive sense. The relations between the two systems are given from Fig. 2 , after eliminating the flight path angle φ, between them as:

$$\vec{R}=\frac{(1+e\cos f)}{\sqrt{1+e^2+2e\cos f}}\vec{T}-\frac{e\sin f}{\sqrt{1+e^2+2e\cos f}}\vec{N}$$

$$\vec{S}=\frac{e\sin f}{\sqrt{1+e^2+2e\cos f}}\vec{T}-\frac{1+e\cos f}{\sqrt{1+e^2+2e\cos f}}\vec{N}$$

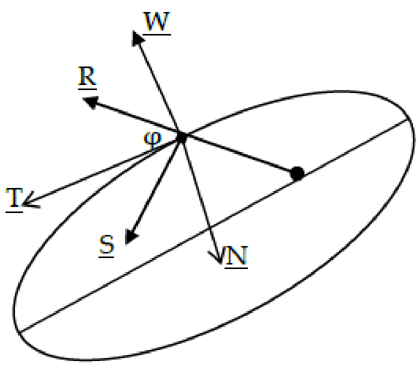

Fig. 2: Transformation from R and S to T and N

The VOP equations in the TNW- coordinate system will take the form:

$$\left(\frac{da}{dt}\right)_D=-\frac{B_c\rho V_S^2}{n\sqrt{1-e^2}}\sqrt{1+e^2+2e\cos f}$$

$$\left(\frac{de}{dt}\right)_D=-\frac{B_c\rho V_S^2\sqrt{1-e^2}}{na}\frac{(e+\cos f)}{\sqrt{1+e^2+2e\cos f}}$$

$$\left(\frac{d\omega}{dt}\right)_D=-\frac{B_c\rho V_S^2\sqrt{1-e^2}}{nae}\frac{\sin f}{\sqrt{1+e^2+2e\cos f}}$$

$$\left(\frac{dM}{dt}\right)_D=n-\frac{B_c\rho V_S^2}{na}\frac{e(1-e)\sin f}{\sqrt{1+e^2+2e\cos f}}\left(\frac{1}{(1-e)^2+\sqrt{(1-e)}}+\frac{1}{(e+\cos f)}\right) \quad (4.13)$$

We have to express the satellite velocity v_s^2 in terms of the orbital parameters, where we take the velocity of earth $\omega_E=0$.

$$V_S^2=\dot{r}^2+r^2\dot{f}^2 \quad (4.14)$$

and we have

$$\dot{r}=\frac{r^2\dot{f}}{a(1-e^2)}e\sin f=\frac{h}{a(1-e^2)}e\sin f \quad (4.15)$$

(h is the angular momentum).

Then we can write equation (4.14) in the form

$$V_S^2=\frac{\mu}{a(1-e^2)}(1+2e\cos f+e^2) \quad (4.16)$$

Substituting equation (4.16) into equations (4.13) we have the variations of the orbital elements due to drag

$$\left(\frac{da}{dt}\right)_D=-\left(\frac{\mu B_c\rho}{na}\right)\frac{\left(1+e^2+2e\cos f\right)^{3/2}}{\left(1-e^2\right)^{3/2}}$$

$$\left(\frac{de}{dt}\right)_D=-\left(\frac{\mu B_c\rho}{na^2}\right)\left(\frac{e+\cos f}{\sqrt{1-e^2}}\right)\sqrt{1+e^2+2e\cos f}$$

$$\left(\frac{d\omega}{dt}\right)_D=-\left(\frac{\mu B_c\rho}{na^2e}\right)\left(\frac{e\sin f}{\sqrt{1-e^2}}\right)\sqrt{1+e^2+2e\cos f}$$

$$\left(\frac{dM}{dt}\right)_D=n-\left(\frac{\mu B_c\rho}{na^2e}\right)\left(\frac{e\sin f}{(1-e^2)+\sqrt{1-e^2}}-\frac{1}{1+e\cos f}\right)\sqrt{1+e^2+2e\cos f} \quad (4.17)$$

Next, to integrate we must change the independent variable from time to the true anomaly, via the transformation .

$$\frac{dt}{df}=\frac{r^2}{na^2\sqrt{1-e^2}}=\frac{(1-e^2)^{3/2}}{n(1+e\cos f)^2} \quad (4.18)$$

Substituting with (4.21) into system (4.20), the first equation gives

$$\frac{da}{df} = -\rho B_c a^2 \frac{\left(1 + 2e\cos f + e^2\right)^{\frac{3}{2}}}{\left(1 + e\cos f\right)^2} \tag{4.19}$$

$$\frac{de}{df} = -\rho B_c a^2 \frac{\left(1 + 2e\cos f + e^2\right)^{\frac{1}{2}}\left(e + \cos f\right)}{\left(1 + e\cos f\right)^2} \tag{4.20}$$

The presence of $\cos f \cos f$, means that there are periodic variations. So, again let's change the true anomaly to the eccentric anomaly, using the transformations

$$\cos f = \frac{\cos E - e}{1 - e\cos E}, \quad \frac{df}{dE} = \frac{a\sqrt{1-e^2}}{r} \tag{4.21}$$

$$\frac{da}{dE} = \frac{da}{df}\frac{df}{dE} = -\rho B_c a^2 \frac{\left(1 + e\cos E\right)^{\frac{3}{2}}}{\sqrt{1 - e\cos E}} \tag{4.22}$$

$$\frac{de}{dE} = \frac{de}{df}\frac{df}{dE} = -\rho B_c a^2 \left(\cos E\right)\left(\frac{1 + e\cos E}{1 - e\cos E}\right)^{\frac{3}{2}} \tag{4.23}$$

We can now consider the air density $\rho\rho$ as:

$$\rho = \rho_0 e^{-(\eta - \eta_o)/H} \tag{4.24}$$

where

ρ_0 is the air density at perigee,

η is the satellite altitude,

η_0 is the altitude at the perigee, and

H is the scale height.

Since $\eta = r - R_e$ and

$\eta_0 = a\left(1 - e\right) - R_e$

Then

$$\eta - \eta_0 = r - a\left(1 - e\right) = ae\left(1 - \cos E\right)$$

Thus, we can put the density model into the form

$$\rho = \rho_0 e^{-k(1 - \cos E)} \tag{4.25}$$

where $k = \dfrac{a_e}{H}$,

Expanding the density function up to O(k^3)

$$\rho = \frac{\rho_0}{24} e^{-k}(24 + 6k^2 + 24k\cos(E) + 3k^3\cos(E) + 6k^2\cos(2E) + k^3\cos(3E) \tag{2.26}$$

Substituting equation (4.26) into equation (4.23) and then into equations (4.22), we can calculate the integrations from 0 to 2π, after using the relation $\mu = n^2 a^3$ and get the results as:

$$\bar{a}_D = -\rho B_C a^2 e^{-k}\left[1 + \frac{3}{4}e^2 + \frac{a^2 e^2}{4H^2} + \frac{a^2}{H}\right] \tag{4.27}$$

$$\bar{e}_D = -\rho B_C a^2 (1 - e^{-2})^2\left[\frac{3}{2}(4e + 3e^2) + \frac{3}{2}(8 + 7e^2)\frac{a}{4H} + \frac{7}{2}\frac{a^2 e^3}{H^2} + \frac{3}{2}\frac{a^3 e^3}{H^3}\right] \tag{4.28}$$

These equations represent the secular changes in the orbit due to drag force

5. ADDING PERTURBATIONS

$$a(t) = a_0 + \left(\Delta a\right)_D$$

$$e(t) = e_0 + \left(\Delta e\right)_D$$

$$\omega(t) = \omega_0 + \left(\Delta\omega\right)obl$$

$$\Omega(t) = \Omega_0 + \left(\Delta\Omega\right)_{obl}$$

$$M(t) = M_0 + \left(\Delta M\right)_{obl}$$

6. RESULTS AND CONCLUSION

6.1. Test orbits

We will take as an example the following two satellites have the two-line elements are (http://celestrak.com).

6.2. Numerical results

We apply the above algorithm on satellites to get the intervals of visibility between two satellites.

we studied the intervals visibility times for about one day (1450 minutes) under forces mentioned above and compare between two cases first Oblateness only and second Oblateness with drag force

The following table has 15.56 periods of short period satellite (EGYPTSAT and TRMM) and shows 38 time intervals of visibility between two satellites.

The intervals 12, 16, 20, 24, 27, 29, 31 and 33 started at end of periods and continued to following periods.

Table(1): the six Orbital elements of satellites

NORAD Two- Line ELEMENT Sets						
Satellite	Orbital Elements					
Epoch Year & Julian Date	i	Ω	e	ω	M	n (rev/d)
1 – RS-40 15225.76240299	82.4736	24.0715	0.0017574	56.7063	303.5694	12.42569673
2- PROITERES 15225.75945562	98.2236	326.0322	0.0013327	111.8177	248.4451	14.75889107
3- EGYPTSAT 1 08142.74302347	98.0526	218.7638	0.0007144	61.2019	298.9894	14.69887657
4 - TRMM 08141.84184490	34.9668	53.6865	0.0001034	61.2019	88.9226	15.558752725

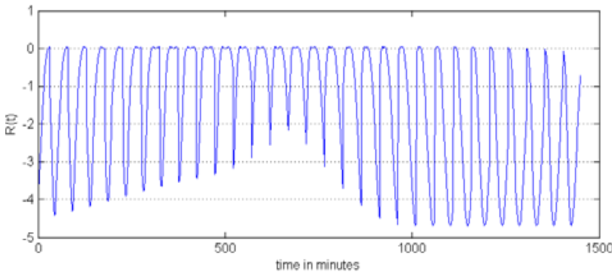

Figure 2:visibililty intervals Between EGYSAT1 & TRMM under Oblateness

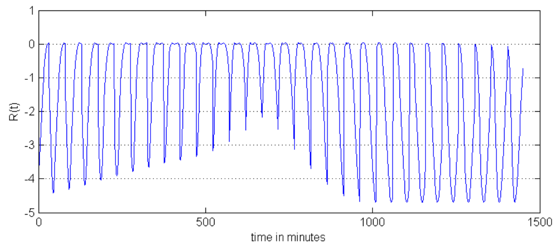

Figure 2:visibililty intervals Between EGYSAT1 & TRMM under Oblateness

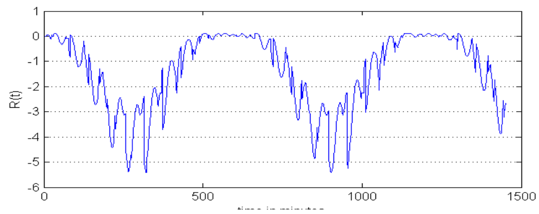

Figure 3:visibililty intervals Between RS-40 & Poriteres under Oblateness

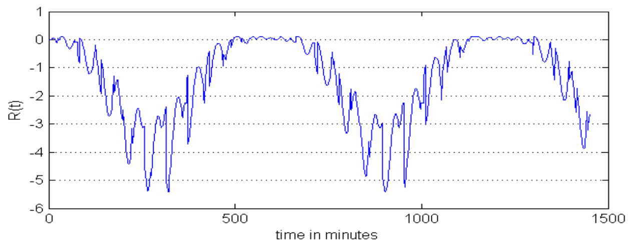

Figure 4:visibililty intervals Between RS-40 & Poriteres under Drag and Oblateness

*The most of intervals (36) don't change in comparing between Oblateness only and (Drag & Oblateness), these intervals are same in the time of rise and set

*The only two intervals which are intervals number (14 and 24) changed between two cases.

*The change in these two intervals is by decreasing the interval of visibility according to applying the drag force, but this decreases is small by amount one minute only in the interval of visibility.

*The minute which decreases in the long of intervals (14 and 24) is in the end of the interval number 14, but at the start of the interval 24.

*The period three has more than seven intervals (5 to 11) and interval number 12 starts in this period.

*After the interval 38 of visibility in period number 14 to the end of the periods there is no visibility.

*The difference in start and end of visibility intervals was expected, acceptable and means good accuracy for our calculations which become from the changes in perigee and apogee times for the satellites according to applied forces .

*The following table (3) has 14.75 periods of short period satellite (PROITERES) and shows 18 time intervals of visibility between two satellites(RS-40and PROITERES) according Oblateness and both of Drag and Oblateness together

*There are 14 intervals don't change in comparing between Oblateness only and (Drag &

Table(2): **shows intervals of visibility (rise-set intervals) and compares between two cases (Oblateness in column three and both of Oblateness and drag column four) for the 38 intervals of visibility as column two in the 16 periods of TRMM per one day.**

Time	intervals	Oblateness			Drag&Oblateness			Period of TRMM
		Rise	Set	Intervals with minutes	Rise	set	Intervals with minutes	
Start at 20 July 2008 20:12:15.4	1	28	31	3	28	31	3	1
	2	74	80	6	74	80	6	
	3	120	129	9	120	129	9	2
	4	166	176	10	166	175	10	
	5	213	222	9	213	222	9	3
	6	225	227	2	225	227	2	
21 July 2008	7	259	268	9	259	268	9	
	8	272	276	4	272	276	4	
	9	305	314	9	305	314	9	
	10	318	325	7	318	325	7	
	11	351	360	9	351	360	9	
	12	364	-373	9	364-	-373	9	
	13	398	407	9	398	407	9	4
	14	410	420	10	410	419	9	
	15	444	453	9	444	453	9	
	16	457	-	9	457	-	9	
		-	466		-	466		5
	17	491	499	8	491	499	8	
	18	503	512	9	503	512	9	
	19	538	545	7	538	545	7	
	20	549	-	9	549	-	9	
		-	558		-	558		6
	21	585	592	7	586	592	6	
	22	595	604	9	595	604	9	
	23	633	638	5	633	638	5	
	24	641	-	10	642	-	9	
			651			651		7
	25	681	685	4	681	685	4	
	26	688	697	9	688	697	9	
	27	728	-	15	728	-	15	
		-	743		-	743		8
	28	776	789	13	776	789	13	
	29	824	-	12	824	-	12	
		-	836		-	836		9
	30	872	882	10	872	882	10	
	31	919	-	10	919	-	10	
		-	929		-	929		10
	32	967	977	10	967	977	10	
	33	1015	-	9	1015	-	9	
			1024			1024		11
	34	1063	1071	8	1063	1071	8	
	35	1112	1118	6	1112	1118	6	12
	36	1160	1165	5	1160	1165	5	
	37	1208	1212	4	1208	1212	4	13
	38	1257	1259	2	1257	1259	2	14

Table(3): shows intervals of visibility (rise-set intervals) and compares between two cases (Oblateness in column three and both of Oblateness and drag column four) for the 18 intervals of visibility as column two in the 15 periods of PROITERES per one day.

Time	intervals	Oblateness			Drag&Oblateness			Period of PROI-TERES
		Rise	Set	Intervals with minutes	Rise	set	Intervals with minutes	
Start at 15 August 2015 18:13:15.37	1	7	17	10	7	17	10	1
	2	25	43	18	25	43	18	
	3	83	87	4	83	87	4	
16 August 2015	4	495	515	20	495	515	20	5
	5	518	521	3	518	521	3	
	6	538	-	71	538	-	71	
		-	609		-	609		6
	7	615	632	17	615	632	17	
	8	646	653	7	646	653	7	
	9	663	677	14	663	677	14	
	10	1090	1093	3	1090	1092	2	11
	11	1123	1125	2	1124	1125	1	
	12	1128	1162	34	1128	1162	34	
	13	1172	1192	20	1173	1192	19	12
	14	1196	1239	43	1196	1239	43	
	15	1242	1247	5	1242	1247	5	
	16	1250	1266	16	1250	1266	16	
	17	1287	1288	1	1287		<1	13
21 July 2008	18	1300	1310	10	1300	1310	10	

Oblateness), these intervals are same in the time of rise and set

*The anther four intervals which are intervals number (10,11,13 and 17) changed between two cases.

*The change in these four intervals is by decreasing the interval of visibility according to applying the drag force, but this decreases is small by amount one minute only in the interval of visibility.

*The intervals (10 and 17) are lost thus minute at the end of interval which end earlier under effect of the Drag force, to be less than one minute in the interval number 17 after applying the Drag force in sated of two minutes before it, and two minutes in interval number 10 opposite to three minutes without Drag.

*The intervals (11 and 13) start later by one minute after applying the Drag force than Oblateness only.

*The periods (2upto 4 and 7upto 10) did not include any interval of visibility, which repeated again by the end of interval 18 in period number 13 to the end of the periods.

the interval 6 started at end of period number 5 and continued to following periods.

In general:

*The difference in start and end of visibility intervals was expected, acceptable and means good accuracy for our calculations which become from the changes in perigee and apogee times for the satellites according to applied forces .

*The more effect of Drag force observed

in the visibility between satellites RS-40 and PROITERES than the satellites EGYPTSAT and TRMM because the first two satellites at latitude which under the effect of the Drag force but the EGYPTSAT is at the end of this area.

6. REFERENCES

[1] Escobal,P.R. “Rise and set Times of a Satellite about an Oblate Planet,” AIAA Journal Vol. I, No. 10, October 1963, pp.2306-2310.

[2] Escobal,P.R. “Methods of orbit determination, Robert E. Krieger Publishing Company, Huntington, New York, 1965, pp. 26-29 and 360 – 369.

[3] LAWTON, J. A. “Numerical Method for Rapidly Determining Satellite – Satellite and Satellite-Ground Station In – View Periods, “ Journal of guidance, Navigation, and Control, vol. 10, January-February, 1987. pp.32-36.

[4] Salvatore Alfano , David Negron, Jr., and Jennifer L. Moore “Rapid Determination of Satellite Visibility Periods”, The Journal of the Astronautical Sciences, Vol.40, No.2, April-June 1992, pp.281-296.

[5] P. L. Palmer & Yan Mai “A Fast Prediction Algorithm of Satellite Passes”, 14th Annual AIAA/USU Conference on Small Satellite, Surrey Space Centre University of Surrey,Guildford,GU2 7XH, UK]

[6] Vallado, D.A. “Fundamental of Astrodynamics and Application,” 3rd edition, Springer, New York, 2007, pp. 159-176.

[7] Michel Capderou, Satellites, Orbits and Missions, Springer, New York, 2005, pp. 65-71.

[8] Roy, A. E. 2005. Orbital Motion. Adam Hilger, J. W., Arrow smith, Ltd., Bristoll, England.

[9] Roy, A. E. 1988. Orbital Motion. Adam Hilger, J. W., Arrow smith, Ltd., Bristoll, England.