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## TESTING EXPONENTIALITY AGAINST EXPONENTIAL BETTER THAN USED IN LAPLACE TRANSFORM ORDER BASED ON GOODNESS OF FIT APPROACH

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### ABSTRACT

In this paper, test statistic for testing exponentiality versus exponential better than used in Laplace transform order based on goodness of fit approach is discussed. Pitman's asymptotic efficiencies of our test are calculated and compared with other tests. The percentiles of this test are tabulated. The powers of the test are estimated for famously used distributions in aging problems. In case of censored data our test is applied and the percentiles are calculated and tabulated. Finally, examples in different fields are used as practical applications for the proposed test.

Keywords: Goodness of fit, censored data, Monte Carlo simulation, EBUL.

### 1 INTRODUCTION

The notion of ageing is very important in reliability analysis. "No ageing" means that the age of a component has no influence on the rest of the lifetime of the component. Concepts of ageing describe how a component or system improves or deteriorates with age. Many classes of life distributions are defined in the literature according to their ageing properties, see for example Barlow and Proschan (1981), Abouammoh and Ahmad (1992) and El-Batal (2002). An important aspect of such classifications is that the exponential distribution is nearly always a member of each class which have the property of memory less "as good as new" and its have a constant failure rate, these features increased the importance of the exponential distribution. Testing exponentiality against different classes of life distributions has good attention from many researchers. For testing against increasing failure rate (IFR), see Proschan and Pyke (1967), Barlow (1968) and Ahmad (1975), among others. For testing against new better than used (NBU), see Hollander and Proschan (1972), Koul (1977), Kumazawa (1983) and Ahmad (1994). For testing against new better than used in Laplace transform order (NBUL) and exponential better than used (EBU), see Diab et al (2009) and El-Batal (2002). For relationship between certain classes of life distributions see Ravi and Prathibha (2012).

Definition 1.1 A life distribution  $F$ , with  $F(0) = 0$ , survival function  $\bar{F}$  and finite mean  $\mu$  is said to be EBU if

$$\bar{F}(x+t) \leq \bar{F}(t)e^{-\frac{x}{\mu}}, x, t > 0, \quad (1.1)$$

or

$$\bar{F}_t \leq e^{-\frac{x}{\mu}}, x, t > 0.$$

Where  $\bar{F}_t = \frac{\bar{F}(x+t)}{\bar{F}(t)}$  represents the survival function related to the random residual life time  $X_t$

Note that, the above definition which is introduced by El-Batal (2002), is motivated by comparing the life length  $X_t$  of a component of age  $t$  with another new component of life length  $X$  which is exponential with the same mean  $\mu$ . Recently a new class of life distribution named exponential better than used in Laplace transform order (EBUL) is introduced by Mahmoud et al (2014) which expand the EBU class.

Definition 1.2  $X$  is said to be EBUL if

$$\int_0^{\infty} e^{-sx} \bar{F}(x+t) dx \leq \frac{\mu}{(\mu s + 1)} \bar{F}(t) \quad x, t > 0, s \geq 0.$$

(1.2)

Testing exponentiality based on goodness of fit approach against many classes of life distributions was studied by some authors such as Abu-Youssef (2009), Kayid et al (2010), Ismail and Abu-Youssef (2012) and Mahmoud and Rady (2013). In the current study, A goodness of fit approach is used to testing exponentiality versus (EBUL). In Section 2 based on U- statistic our test is developed and its asymptotic properties are studied. In that section, Monte Carlo

null distribution critical points are simulated for samples size  $n = 5(5)35, 39, 40(5)50$  and the power estimates are also calculated and tabulated. In Section 3, we dealing with right-censored data and selected critical values are tabulated. Finally, in Section 4 we discuss some applications to demonstrate the utility of the proposed test in reliability analysis.

## 2 Testing Against EBUL Alternatives

In this section a test statistic is constructed to test exponentiality against (EBUL) based on goodness of fit approach. The following lemma is needed.

**Lemma 2.1** If  $X$  a random variable with distribution function  $F$  and  $F$  belongs to EBUL class, then

$$(s\mu + 1)(1 - \phi(s)) \leq s(\mu + 1)(1 - \phi(1)), \quad s \geq 0, \quad (2.1)$$

$$\text{where } \phi(s) = E^{-sX} = \int_0^\infty e^{-sx} f(x) dx$$

Proof.

Since  $F$  is EBUL then,

$$\int_0^\infty e^{-sx} \bar{F}(x+t) dx \leq \frac{\mu}{(\mu s + 1)} \bar{F}(t) \quad x, t > 0.$$

Consider the following integral

$$\int_0^\infty \int_0^\infty e^{-t} e^{-sx} \bar{F}(x+t) dx dt \leq \frac{\mu}{(\mu s + 1)} \int_0^\infty e^{-t} \bar{F}(t) dt. \quad (2.2)$$

Setting

$$\begin{aligned} I_1 &= \int_0^\infty e^{-t} \bar{F}(t) dt = E \int_0^\infty e^{-t} I(X > t) dt \\ &= E \int_0^X e^{-t} dt = (1 - E^{-X}) \end{aligned}$$

it is easy to show that

$$I_1 = (1 - \phi(1)) \quad (2.3)$$

Setting

$$I_2 = \int_0^\infty \int_0^\infty e^{-t} e^{-sx} \bar{F}(x+t) dx dt.$$

So  $I_2$  can be put in the following form

$$I_2 = \int_0^\infty \int_0^\infty e^{-v} e^{-s(u-v)} \bar{F}(u) du dv$$

$$\begin{aligned} &= \int_0^\infty \int_0^u e^{-u(1-s)} e^{-v} \bar{F}(v) du dv \\ &= \frac{1}{1-s} \left[ \int_0^\infty e^{-v} \bar{F}(v) dv - \int_0^\infty e^{-v} \bar{F}(v) dv \right] \end{aligned}$$

therefore

$$I_2 = \frac{1}{1-s} \left[ \frac{1}{s} (1 - \phi(s)) - (1 - \phi(1)) \right]. \quad (2.4)$$

Substituting (2.3) and (2.4) into (2.2), we get

$$(s\mu + 1)(1 - \phi(s)) \leq s(\mu + 1)(1 - \phi(1))$$

This completes the proof.

Let  $X_1, \dots, X_n$  denote a random sample from a distribution  $F$ , we wish to test  $H_0: F$  is exponential against  $H_1: F$  is EBUL and not exponential. By using the following as a measure of departure from  $H_0$  in favor of  $H_1$

$$\zeta(s) = s(\mu + 1)(1 - \phi(1)) - (s\mu + 1)(1 - \phi(s))$$

note that, under  $H_0$   $\zeta(s) = 0$ , while it is positive under  $H_1$ . Define the test statistic  $\zeta_n(s)$  as follows

$$\zeta_n(s) = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n [s(1 + X_i)(1 - e^{-X_i}) - (1 + X_i)(1 - e^{-X_i})]$$

To make the test invariant, let

$$\Delta_n(s) = \frac{\zeta_n(s)}{\bar{X}^2},$$

where  $\bar{X} = \sum_{i=1}^n \frac{X_i}{n}$  is the sample mean.

Then

$$\Delta_n(s) = \frac{1}{n^2 \bar{X}^2} \sum_i \sum_j \phi(X_i, X_j) \quad (2.5)$$

where

$$\phi(X_i, X_j) = s(1 + X_i)(1 - e^{-X_i}) - (1 + X_i)(1 - e^{-X_i}) \quad (2.6)$$

The following theorem summarizes the asymptotic properties of the test.

**Theorem 2.2** As  $n \rightarrow \infty$ ,  $(\Delta_n(s) - \zeta(s))$  is asymptotically normal with mean 0 and variance  $\sigma^2(s)/n$ , where  $\sigma^2(s)$  is given in (2.7). Under  $H_0$ , the variance reduces to (2.8).

$$\sigma^2 = V\{E[\phi(X_1, X_2) | X_1] + E[\phi(X_1, X_2) | X_2]\}.$$

Remember the definition of  $\phi(X_i, X_j)$  in (2.6), thus it is not difficult to show that

$$E[\phi(X_1, X_2) | X_1] = s - 1 - s \int_0^\infty e^{-x} dF(x) + \int_0^\infty e^{-x} dF(x) + s \int_0^\infty e^{-x} dF(x) - s \int_0^\infty e^{-x} dF(x)$$

Similarly,

$$E[\phi(X_1, X_2) | X_2] = s - 1 - s^{-x} + e^{-x} + s^{-x} \int_0^\infty x dF(x) - s^{-x} \int_0^\infty x dF(x)$$

Hence

$$\sigma^2(s) = Var\{2s - 2 - s \int_0^\infty e^{-x} dF(x) + \int_0^\infty e^{-x} dF(x) + s \int_0^\infty e^{-x} dF(x) - s \int_0^\infty e^{-x} dF(x) - s^{-x} + e^{-x} + s^{-x} \int_0^\infty x dF(x) - s^{-x} \int_0^\infty x dF(x)\}. \tag{2.7}$$

Under  $H_0$

$$\sigma_0^2(s) = \frac{s^2(s-1)^2(2s^2+s+2)}{2(2s+1)(s+2)(s+1)^2}. \tag{2.8}$$

### 2.1 Asymptotic efficiency

To decide the quality of this procedure, we compare its Pitman asymptotic efficiencies (PAE) with some other tests in Table (1) for the following alternative distributions.

- 1) The Weibull distribution:

$$\bar{F}_1(x) = e^{-x^\theta}, x \geq 0, \theta \geq 1.$$

- 2) The linear failure rate distribution (LFR):

$$\bar{F}_2(x) = e^{-x - \frac{\theta}{2}x^2}, x \geq 0, \theta \geq 0.$$

- 3) The Makeham distribution:

$$\bar{F}_3(x) = e^{-x - \theta(x + e^{-x} - 1)}, x \geq 0, \theta \geq 0.$$

Note that when  $\theta = 1$  and  $\theta = 0$ , the (i), (ii) and (iii) distributions reduce to the exponential distribution.

The PAE is defined by:

$$PAE(\Delta_n(s)) = \frac{1}{\sigma_0(s)} \left| \frac{d}{d\theta} \zeta(s) \right|_{\theta \rightarrow \theta_0}$$

This leads to:

$$PAE[\Delta_n(0.2), Weibull] = 7.34682,$$

$$PAE[\Delta_n(0.2), LFR] = 4.19399$$

and

$$PAE[\Delta_n(0.2), Makeham] = 1.71572.$$

Table (1) Comparison between the PAE of our test and some other tests

| Test                          | Weibull | LFR     | Makeham |
|-------------------------------|---------|---------|---------|
| Kango (1993)                  | 0.132   | 0.433   | 0.144   |
| Mugdadi and Ahmad (2005)      | 0.170   | 0.408   | 0.039   |
| Mahmoud and Abdul Alim (2008) | 0.405   | 0.433   | 0.289   |
| Our test                      | 7.34682 | 4.19399 | 1.71572 |

It is clear that our test has the greatest efficiency in all cases.

### 2.2 Monte Carlo null distribution critical values

In this subsection Monte Carlo null distribution critical points for our test  $\Delta_n(0.2)$  are simulated based on 10000 generated samples from the standard exponential distribution using Mathematica 8 program. Table (2) gives the upper percentile points of  $\Delta_n(0.2)$ , where  $n = 5(5)35, 39, 40(5)50$ .

Table (2) The upper percentile points of  $\Delta_n(0.2)$  with 10000 replications

| n  | 90%        | 95%        | 99%       |
|----|------------|------------|-----------|
| 5  | 0.00654827 | 0.00831552 | 0.0117857 |
| 10 | 0.00680458 | 0.00866917 | 0.0123706 |
| 15 | 0.00724927 | 0.00918529 | 0.013069  |
| 20 | 0.00802681 | 0.0101987  | 0.0142138 |
| 25 | 0.00890195 | 0.0111627  | 0.0159395 |
| 30 | 0.00986511 | 0.0125176  | 0.017838  |
| 35 | 0.0111477  | 0.0139494  | 0.0194316 |
| 39 | 0.012984   | 0.0163702  | 0.0227707 |
| 40 | 0.0133555  | 0.0165929  | 0.023059  |
| 45 | 0.0174138  | 0.0221825  | 0.0304237 |
| 50 | 0.0273877  | 0.0331176  | 0.043624  |

From Table (2), it is obvious that the critical values are increasing as the samples size increasing and they are increasing as the confidence levels increasing.

**2.3 The power of test**

In this subsection the power of our test  $\Delta_n(0.2)$  will be estimated at significance level  $\alpha = 0.05$  with respect to two alternatives Weibull and linear failure rate (LFR) distributions based on 10000 samples. Table (3) gives the power estimates with parameter  $\theta = 2, 3$  and  $4$  at  $n = 10, 20$  and  $30$

**Table(3) The power estimates of the Statistic  $\Delta_n(0.2)$**

| n  |   | Weibull | LFR    |
|----|---|---------|--------|
| 10 | 2 | 0.9928  | 0.8524 |
|    | 3 | 1.0000  | 0.9037 |
|    | 4 | 1.0000  | 0.926  |
| 20 | 2 | 0.9991  | 0.869  |
|    | 3 | 1.0000  | 0.9282 |
|    | 4 | 1.0000  | 0.9563 |
| 30 | 2 | 0.9987  | 0.8353 |
|    | 3 | 1.0000  | 0.9266 |
|    | 4 | 1.0000  | 0.9604 |

It is obviously that the power estimates increase as the sample size increases for each value of  $\theta$ .

**3 Testing for Censored Data**

Here, a test statistic is proposed to test  $H_0$  versus  $H_1$  with randomly right-censored data. Such a censored data is usually the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modeled as follows. Suppose  $n$  objects are put on test, and

$X_1, X_2, \dots, X_n$  denote their true life times. Let

$X_1, X_2, \dots, X_n$  be independent and identically distributed (i.i.d) according to a continuous life distribution  $F$ . Let  $Y_1, Y_2, \dots, Y_n$  be (i.i.d) according to a continuous life distribution  $G$ . Also we assume that  $X$ 's and  $Y$ 's are independent. In the randomly right-censored model, the pairs  $(Z_j, \delta_j)$ ,  $j = 1, \dots, n$ , are observed, where  $Z_j = \min(X_j, Y_j)$  and

$$\delta_j = \begin{cases} 1, & \text{if } Z_j = X_j \text{ (j - th observation is uncensored)} \\ 0, & \text{if } Z_j = Y_j \text{ (j - th observation is censored)} \end{cases}$$

Let  $Z_{(1)} = Z_{(1)} < Z_{(2)} < \dots < Z_{(n)}$

$Z_{(j)}$  denote the orderd  $Z$ 's and  $\delta_{(j)}$  is  $\delta_j$  corresponding to  $Z_{(j)}$ . Using the censored data  $(Z_{(j)}, \delta_{(j)})$ ,  $j = 1, \dots, n$ . Kaplan and Meier (1958) proposed the product limit estimator, as follows

$$\bar{F}_n(X) = \prod_{j: Z_{(j)} \leq X} \{ (n-j)(n-j+1)^{\delta_{(j)}} \}, X \in [0, Z_{(n)}]$$

Now, for testing  $H_0: \zeta_c(s) = 0$  against  $H_1: \zeta_c(s) > 0$ , using the randomly right censored data, the following test statistic is proposed

$$\zeta_c(s) = s(\mu + 1)(1 - \phi(1)) - (s\mu + 1)(1 - \phi(s))$$

For computational purpose,  $\zeta_c(s)$  may be rewritten as

$$\zeta_c(s) = s(\Omega + 1)(1 - \tau) - (s\Omega + 1)(1 - \eta)$$

where

$$\eta = \sum_{j=1}^n e^{-sZ_{(j)}} \left[ \prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)} \right], \Omega = \sum_{k=1}^n \left[ \prod_{m=1}^{k-1} C_m^{\delta(m)} (Z_{(k)} - Z_{(k-1)}) \right]$$

$$\tau = \sum_{j=1}^n e^{-Z_{(j)}} \left[ \prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)} \right]$$

and

$$\bar{F}_n(Z_j) = \bar{F}_n(Z_{j-1}) - \bar{F}_n(Z_j) c_k = [n - k][n - k + 1]^{-1}$$

To make the test invariant, let

$$\Delta_c(s) = \frac{\zeta_c(s)}{\bar{Z}^2}, \text{ where } \bar{Z} = \sum_{i=1}^n \frac{Z_{(i)}}{n}. \quad (3.1)$$

**3.1 Monte Carlo null distribution critical values in censored case**

In this subsection the monte carlo null distribution critical values of  $\Delta_c$  at  $s = 0.2$  for samples sizes  $n = 20, 25, 30, 40, 50, 51, 60, 70, 81$  with 10000 replications are simulated from the standard exponential distribution by using Mathematica 8 program. Table (4) gives the upper percentile points of the statistic  $\Delta_c$ .

**Table (4) The Upper Percentile Points of  $\Delta_c$**

| n  | 90%      | 95%                        | 99%                        |
|----|----------|----------------------------|----------------------------|
| 20 | 0.341634 | 68.1421                    | 166861.0                   |
| 25 | 3.47767  | 1584.12                    | 2.29304 * 10 <sup>7</sup>  |
| 30 | 8.33774  | 6179.51                    | 2.16501 * 10 <sup>9</sup>  |
| 40 | 39.4047  | 421367.0                   | 1.77345 * 10 <sup>12</sup> |
| 50 | 329.65   | 4.18467 * 10 <sup>7</sup>  | 6.42456 * 10 <sup>15</sup> |
| 51 | 152.416  | 6.99417 * 10 <sup>7</sup>  | 1.09826 * 10 <sup>17</sup> |
| 60 | 959.563  | 2.56878 * 10 <sup>9</sup>  | 1.81463 * 10 <sup>20</sup> |
| 70 | 8822.87  | 1.19424 * 10 <sup>11</sup> | 6.36828 * 10 <sup>23</sup> |
| 81 | 18134.2  | 6.13836 * 10 <sup>12</sup> | 1.36052 * 10 <sup>27</sup> |

Table (4) shows that the critical values increase as the sample size and the confidence level increase.

4 Some Applications

In this section, we apply our test on some real data-sets at 95% confidence level.

4.1 Case of non censored data

In this section two examples are presented considering  $s = 0.2$ .

**Example 4.1.1** Consider the data in Mahmoud et al (2005) which represent 39 liver cancers patients taken from Elminia cancer center Ministry of Health -- Egypt, which entered in (1999). The ordered life times (in days)

|    |    |    |     |     |     |     |     |     |    |
|----|----|----|-----|-----|-----|-----|-----|-----|----|
| 10 | 14 | 14 | 14  | 14  | 14  | 15  | 17  | 18  | 20 |
| 20 | 20 | 20 | 20  | 23  | 23  | 24  | 26  | 30  | 30 |
| 31 | 40 | 49 | 51  | 52  | 60  | 61  | 67  | 71  | 74 |
| 75 | 87 | 96 | 105 | 107 | 107 | 107 | 116 | 150 |    |

In this case,  $\Delta_n(0.2) = -0.000260314$  which is less than the critical value in Table (2), then we accept  $H_0$  which states that the data set have exponential property.

**Example 4.1.2** Consider the real data-set given in Grubbs (1971) and have been used in Shapiro (1995). This data set gives the times between arrivals of 25 customers at a facility.

|      |      |      |      |      |
|------|------|------|------|------|
| 1.80 | 2.89 | 2.93 | 3.03 | 3.15 |
| 3.43 | 3.48 | 3.57 | 3.85 | 3.92 |
| 3.98 | 4.06 | 4.11 | 4.13 | 4.16 |
| 4.23 | 4.34 | 4.37 | 4.53 | 4.62 |
| 4.65 | 4.84 | 4.91 | 4.99 | 5.17 |

Since  $\Delta_n(0.2) = -0.000353938$  and this value less than the critical value in Table (2). Then we conclude that this data set have the exponential property.

4.2 Case of censored data

In this section two example are presented considering  $s = 0.2$ .

**Example 4.2.1** Consider the data in Susarla and Vanryzin (1978). These data represent 81

survival times of patients of melanoma. Of them 46 represent whole life times (non-censored data) and the observed values are:

|     |     |     |     |     |     |     |     |     |     |    |    |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|----|
| 13  | 14  | 19  | 19  | 20  | 21  | 23  | 23  | 25  | 26  | 26 | 27 |
| 27  | 31  | 32  | 34  | 34  | 37  | 38  | 38  | 40  | 46  | 50 | 53 |
| 54  | 57  | 58  | 59  | 60  | 65  | 65  | 66  | 70  | 85  | 90 | 98 |
| 102 | 103 | 110 | 118 | 124 | 130 | 136 | 138 | 141 | 234 |    |    |

The ordered censored observations are:

|     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 16  | 21  | 44  | 50  | 55  | 67  | 73  | 76  | 80  | 81  | 86  | 93  |
| 100 | 108 | 114 | 120 | 124 | 125 | 129 | 130 | 132 | 134 | 140 | 147 |
| 148 | 151 | 152 | 152 | 158 | 181 | 190 | 193 | 194 | 213 | 215 |     |

Taking into account the whole set of survival data (both censored and uncensored), and computing the statistic from (3.1).

We get  $\Delta_c(0.2) = 4.92229 \times 10^9$  which is greater than the critical value of Table (4). Then we accept  $H_1$  which states that the data set have EBUL property.

**Example 4.2.2** Consider the data in Mahmoud et al (2005) which represent 51 liver cancers patients taken from Elminia cancer center Ministry of Health -- Egypt, which entered in (1999). Out of these 39 represents non-censored data and the others represents censored data. The ordered life times (in days)

• Non-censored data

|    |    |    |     |     |     |     |     |     |    |
|----|----|----|-----|-----|-----|-----|-----|-----|----|
| 10 | 14 | 14 | 14  | 14  | 14  | 15  | 17  | 18  | 20 |
| 20 | 20 | 20 | 20  | 23  | 23  | 24  | 26  | 30  | 30 |
| 31 | 40 | 49 | 51  | 52  | 60  | 61  | 67  | 71  | 74 |
| 75 | 87 | 96 | 105 | 107 | 107 | 107 | 116 | 150 |    |

• Censored data

|    |    |    |    |    |    |     |     |     |     |     |     |
|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|
| 30 | 30 | 30 | 30 | 30 | 60 | 150 | 150 | 150 | 150 | 150 | 185 |
|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|

Taking into account the whole set of survival data (both censored and uncensored), and computing the statistic using (3.1).

The value of  $\Delta_c(0.2)$  is computed to be  $8.22584 \times 10^{69}$  which is greater than the critical value of Table (4). There is enough to accept  $H_1$  which states that the data set have EBUL property.

### Conclusion

A goodness of fit approach is used to testing exponentiality versus (EBUL). The Pitman asymptotic efficiency of this test is studied. The upper percentiles and the power of the proposed test are calculated and tabulated. In case of censored data the critical values of this test are calculated and tabulated. Our test is applied to some real data. Finally, the proposed test in the two cases seem to be simple.

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